

chapter

3

Applications of
Differential Calculus

Section 3.5 Rates of change

PROJECT MATHS
Text & Tests 7

117

$$s = \text{distance} \quad \triangle \begin{array}{c} D \\ \hline S \Delta T \end{array} \quad a = \frac{v-u}{t} = \frac{\Delta v}{\Delta t}$$

" $\frac{d}{dt}$ " = rate of change (w.r.t. time)

$$\frac{ds}{dt} = \text{velocity / speed } (v)$$

$$\frac{d^2s}{dt^2} = \frac{dv}{dt} = \text{acceleration } (a)$$

Example 1

A body moves in a straight line so that its distance s , in metres, from a fixed point after t secs is given by the equation

$$s = 10 + 27t - t^3.$$

- Find (i) the speed after 2 seconds
 (ii) after how many seconds is the body at rest $V=0$
 (iii) the acceleration after 2 seconds.

distance :	$s = 10 + 27t - t^3$
Speed :	$v = 27 - 3t^2$
acceleration :	$a = -6t$


- | | |
|------------------|---------------------------------------------------------------------------------------------------------------------------------|
| (i) $t=2, v=?$ | $v = 27 - 3(2)^2 = 27 - 3(4) = 15 \text{ m/s}$ |
| (ii) $v=0, t=?$ | $27 - 3t^2 = 0$
$9 - t^2 = 0 \Rightarrow t^2 = 9 \Rightarrow t = \pm 3 \text{ s}$
<i>ie. $t = 3 \text{ s}$</i> |
| (iii) $t=2, a=?$ | $a = -6(2) = -12 \text{ m/s}^2$ |

Example 2

A marketing director of a company found that the revenue, € R , from selling a fixed number of produced items at € P each is given by the formula

$$R = 30P - 2P^2.$$

- (i) Find $\frac{dR}{dP}$ and explain what this means.
 (ii) Calculate $\frac{dR}{dP}$ when $P = 10$.
 (iii) For what selling prices is revenue rising?

(i)	$\frac{dR}{dP} = 30 - 4P$
$P=10$ (ii)	$\Rightarrow \frac{dR}{dP} = 30 - 4(10) = -10$
(iii)	
	$30 - 4P > 0$ $-4P > -30$ $4P < 30$ $P < \frac{30}{4} = 7.5$ $P < \text{€}7.50$

5. The population, P , of a new housing estate t years after a certain date is given by the formula $P = 100(5 + t - 0.25t^2)$.
Find the rate of change of the population after 3 years.

expand

$$P = 500 + 100t - 25t^2$$

$\frac{dP}{dt}$ = rate of change of P

$$\frac{dP}{dt} = 100 - 50t$$

$t = 3$ years $\Rightarrow \frac{dP}{dt} = 100 - 50(3) = -50$

6. The expected assets, $\text{€}M$, of a proposed new company will be given by $M = 200000 + 600t^2 - \frac{200}{3}t^3$, where t is the number of months after the business is set up.
- Find the rate of growth of assets at time t months.
 - Find the rate of growth of assets when $t = 3$.
 - Will the rate of growth of assets be zero at any time?

Is $\frac{dM}{dt}$ = rate of growth?

(i) $\frac{dM}{dt} = 1200t - 200t^2$

$t = 3$ (ii) $\Rightarrow \frac{dM}{dt} = 1200(3) - 200(3)^2$
 $= 3600 - 1800$
 $= 1800$

Will $\frac{dM}{dt} = 0$?

(iii) If $1200t - 200t^2 = 0$
 $\Rightarrow 6t - t^2 = 0$
 $t(6 - t) = 0$
 $\Rightarrow t = 0$, $6 - t = 0$
 ignore, $t = 6$ months