chapter 3

Applications of Differential Calculus

Section 3.6 Related rates of change

- If $\frac{dy}{dx} = \frac{2x}{1}$ then $\frac{dx}{dy} = \frac{1}{2x}$

- Simplify : $\frac{\cancel{dR}}{dt} \cdot \frac{dA}{\cancel{dR}} = \frac{dA}{dt}$

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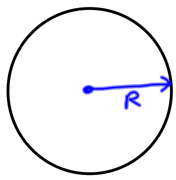
Example 1

The radius of a circle is increasing at the rate of 2 cm/sec.
Find the rate at which the area is increasing when the radius is 3 cm.

Rate of change of Radius $\frac{dR}{dt}$

$\frac{dR}{dt} = 2$



Rate of change of area $\frac{dA}{dt} = ?$

$A = \pi R^2$

$\frac{dA}{dR} = 2\pi R$

$\frac{dA}{dt} = \frac{dA}{dR} \cdot \frac{dR}{dt}$ $= (2\pi R)(2) = 4\pi R$

$R = 3$ $\Rightarrow \frac{dA}{dt} = 4\pi(3) = 12\pi$ cm^2/s

Example 2

The volume, V cm^3, of water in a container is given by the expression $V = 12h^2$, where h cm is the depth of the water. ?
Water is flowing into the container at a steady rate of 90 cm^3/sec.
Find the rate, in cm/sec, at which the depth of the water is increasing when $h = 3$.

Rate of change of h $\frac{dh}{dt} = ?$

$\frac{dV}{dt} = 90$

$V = 12h^2$ $\Rightarrow \frac{dV}{dh} = 24h \Rightarrow \frac{dh}{dV} = \frac{1}{24h}$

$\frac{dh}{dt} = \frac{dh}{dV} \cdot \frac{dV}{dt}$ $\Rightarrow \frac{dh}{dt} = \left(\frac{1}{24h}\right)(90) = \frac{15}{4h}$

$h = 3$ $\Rightarrow \frac{dh}{dt} = \frac{15}{4(3)} = \frac{5}{4}$ cm/s

Exercise 3.6

1. In each of the following, fill in the missing rate:

(i) $\frac{dA}{dt} = \frac{dA}{dr} \cdot \frac{dr}{dt}$ (ii) $\frac{dV}{dr} = \frac{dV}{dt} \cdot \frac{dt}{dr}$ (iii) $\frac{dM}{dt} = \frac{dM}{ds} \cdot \frac{ds}{dt}$

3. If $\frac{dy}{dx} = 10$ and $\frac{dx}{dt} = 2$, find $\frac{dy}{dt}$.

$$\frac{dy}{dt} = \left(\frac{dy}{\cancel{dx}}\right)\left(\frac{\cancel{dx}}{dt}\right) = (10)(2) = 20$$

2. In each of the following, find the indicated rate.

(i) $\frac{dA}{dt} = 8,\ \frac{dA}{dr} = 4,\ \frac{dr}{dt} = ?$

$\frac{dr}{dA} = \frac{1}{4}$

$\frac{dr}{dt} = \frac{dr}{dA} \cdot \frac{dA}{dt}$

$= \left(\frac{1}{4}\right)(8) = 2$

(ii) $\frac{dV}{dt} = 8,\ \frac{dr}{dt} = 2,\ \frac{dV}{dr} = ?$

$\frac{dt}{dr} = \frac{1}{2}$

$\frac{dV}{dr} = \frac{dV}{dt} \cdot \frac{dt}{dr}$

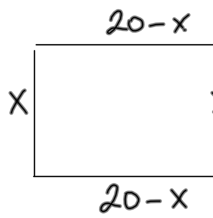
$= (8)\left(\frac{1}{2}\right) = 4$

10. The perimeter of a rectangle has a constant value of 40 cm.
One side, of length x cm, is increasing at the rate of 0.5 cm/sec.

(i) Find, in terms of x, an expression for the area of the rectangle.

(ii) Find the rate at which the area is increasing when $x = 3$ cm.

$\frac{dx}{dt} = \frac{1}{2}$



$P = 40$

$A = LB$

(i) $A = (20-x)x = 20x - x^2$

$\frac{dA}{dt} = ?$

(ii) $\frac{dA}{dx} = 20 - 2x$

$\frac{dA}{dt} = \frac{dA}{dx} \cdot \frac{dx}{dt} = \left(\frac{1}{2}\right)(20-2x) \Rightarrow \frac{dA}{dt} = 10 - x$

$x = 3 \Rightarrow \frac{dA}{dt} = 10 - 3 = 7$