2. Find the roots of the equation $2(x+1)(x-4)-(x-2)^2=0$, leaving your answer in surd form.

$$2\left[x^2-4x+1x-4\right]-\left[x^2-4x+4\right]=0$$

$$2x^2-8x+2x-8-x^2+4x-4=0$$

$$x^2-2x-12=0$$

$$\boxed{\begin{array}{l} a=1 \\ b=-2 \\ c=-12 \end{array}}$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{+2\pm\sqrt{4+48}}{2}$$

$$=\frac{2\pm\sqrt{52}}{2}=\frac{2\pm2\sqrt{13}}{2}=1\pm\sqrt{13}$$

3. Find the range of values of p for which $\underbrace{px^2+2x+1=0}_{\text{QUADRATIC}}$ has no solutions. Root

If no Real Solution ⇒

$$\Delta=b^2-4ac<0$$

DISCRIMINANT

$$\boxed{\begin{array}{l} a=p \\ b=2 \\ c=1 \end{array}}$$

$$2^2-4(p)(1)<0$$

$$4-4p<0$$

$$-4p<-4$$

$$4p>4$$

$$p>1$$

4. Show that the roots of the equation $x^2 - (a + d)x + (ad - b^2) = 0$ are real.

They are real if discriminant is positive

$$\Delta = b^2 - 4ac > 0$$

$a = 1$
$b = -(a+d)$
$c = (ad - b^2)$

$$\Delta = [-(a+d)]^2 - 4(1)(ad - b^2)$$
$$= a^2 + 2ad + d^2 - 4ad + 4b^2$$
$$= \underbrace{a^2 - 2ad + d^2} + 4b^2$$
$$= (a-d)^2 + 4b^2 > 0$$

QED

8. Using the substitution $y = 3^x$, write the equation $3^{2x} - 12(3^x) + 27 = 0$ in terms of y. Hence solve the equation for x.

$$y^2 - 12y + 27 = 0$$
$$(y-9)(y-3) = 0$$
$$y = 9, \quad y = 3$$

$9 = 3^x \Rightarrow x = 2$

$3 = 3^x \Rightarrow x = 1$