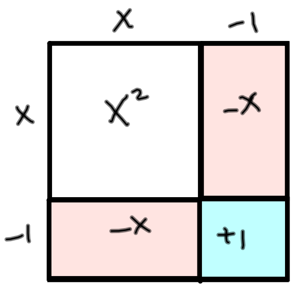
Revision Exercise (Advanced)

1. Express $2x^2 - 4x - 5$ in the form $a(x+h)^2 + k$ and hence,

(i) solve the equation $2x^2 - 4x - 5 = 0$

(ii) find the minimum point of this curve.

Complete the Square



$$2x^2 - 4x - 5 = 2\left[x^2 - 2x - \tfrac{5}{2}\right]$$
$$= 2\left[x^2 - 2x + 1 - 1 - \tfrac{5}{2}\right]$$
$$= 2\left[(x-1)^2 - \tfrac{7}{2}\right]$$
$$= 2(x-1)^2 - 7$$

(i) Solve

$$2x^2 - 4x - 5 = 0$$
$$2(x-1)^2 - 7 = 0$$
$$(x-1)^2 = 7/2$$
$$x - 1 = \pm\sqrt{7/2}$$
$$x = 1 \pm \sqrt{7/2}$$

(ii) min. point of $f(x) = 2(x-1)^2 - 7$ is $(1, -7)$

3. Simplify and then rationalise the denominator of $\dfrac{\sqrt{7}+\sqrt{5}}{\sqrt{80}+\sqrt{5}}$.

$$= \frac{(\sqrt{7}+\sqrt{5})(\sqrt{80}-\sqrt{5})}{(\sqrt{80}+\sqrt{5})(\sqrt{80}-\sqrt{5})} = \frac{\sqrt{7}\sqrt{80} - \sqrt{7}\sqrt{5} + \sqrt{5}\sqrt{80} - \sqrt{5}\sqrt{5}}{80-5}$$

use calculator!

$$= \frac{15+3\sqrt{35}}{75} = \frac{5+\sqrt{35}}{25}$$

5. The motion of a car is given by the equation $8t^2 + 4t = s$, where s is the distance travelled in metres.

(i) By inspection, *estimate* the time, t, taken for the car to pass a point 10 metres away.

(ii) Find, correct to two places of decimals, the time taken and explain why there is only one such time.

(iii) Calculate the percentage error in correcting the answer to two places of decimals.

this question should give a unit for time t   lets say t is in seconds

(i) after 1 second $\Rightarrow$ $s = 8(1)^2 + 4(1) = 12$

So I estimate that it would take a little less than 1 second to travel 10m

(ii) $8t^2 + 4t = 10$ $a = 4$

$4t^2 + 2t - 5 = 0$ $b = 2$

$c = -5$

$$t = \frac{-2 \pm \sqrt{2^2 - 4(4)(-5)}}{2(4)} = \frac{-2 \pm \sqrt{84}}{8}$$

Since the time is positive only

$$\Rightarrow t = \frac{-2 + \sqrt{84}}{8} = \frac{-1 + \sqrt{21}}{4} = 0.895 \simeq 0.90 \text{ seconds}$$

$$\% \text{ ERROR} = \frac{\text{ERROR} \times 100}{\text{RIGHT ANSWER}}$$

$$\text{ERROR} = \text{RIGHT ANSWER} - \text{ROUNDED ANSWER}$$

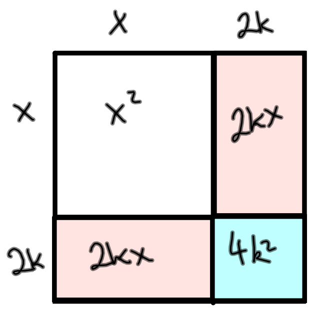
$$\% \text{ ERROR} = \frac{\left(\frac{-1 + \sqrt{21}}{4} - 0.9\right)(100)}{\left(\frac{-1 + \sqrt{21}}{4}\right)}$$

$= 0.486\ \%$

7. Complete the table by stating whether each quantity is positive $(+)^{ve}$ or negative $(-)^{ve}$.

	$k < 0$	$0 < k < \frac{1}{4}$	$k > \frac{1}{4}$
k	*Negative*	+	*Positive*
$4k$	−	+	+
$4k - 1$	−	−	+
$k(4k - 1)$	+	−	+

Using this table, find the range of values of k so that the quadratic expression $x^2 + 4kx + k$ is positive for all values of x.



$x^2 + 4kx + k > 0$

$\underbrace{x^2 + 4kx + 4k^2} - 4k^2 + k > 0$

$(x+2k)^2 > 4k^2 - k$

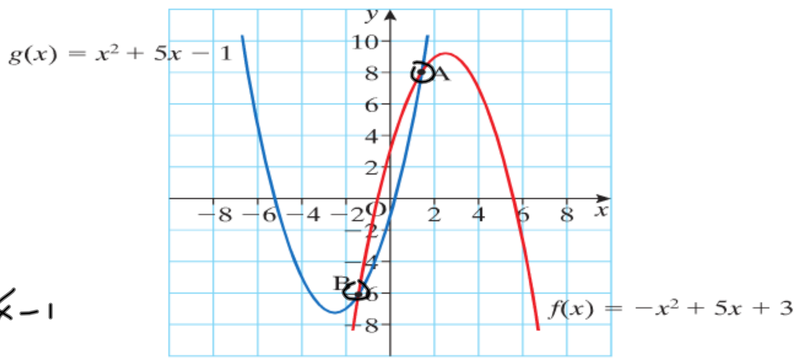
Smallest value is 0

$0 > k(4k-1)$

ie $k(4k-1)$ is negative

$\Rightarrow \quad 0 < k < \frac{1}{4}$

9. Given $f(x) = -x^2 + 5x + 3$ and $g(x) = x^2 + 5x - 1$.
Find the coordinates of the points A and B, leaving your answers in surd form.



$f(x) = g(x)$

$-x^2 + \cancel{5x} + 3 = x^2 + \cancel{5x} - 1$

$-2x^2 = -4$

$x^2 = 2$

$x = \pm\sqrt{2}$

$g(\sqrt{2}) = (\sqrt{2})^2 + 5(\sqrt{2}) - 1$

$= 2 + 5\sqrt{2} - 1 = 1 + 5\sqrt{2}$

$A = (\sqrt{2}, 1 + 5\sqrt{2})$

$g(-\sqrt{2}) = (-\sqrt{2})^2 + 5(-\sqrt{2}) - 1$

$= 2 - 5\sqrt{2} - 1 = 1 - 5\sqrt{2}$

$B = (-\sqrt{2}, 1 - 5\sqrt{2})$

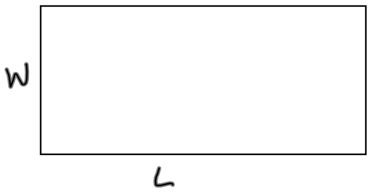
11. If r_1 and r_2 are the roots of the equation $x^2 - \sqrt{3}x - 6 = 0$, evaluate $r_1 r_2$.

Remember...

$X^2 - (\text{Sum of Roots})X + (\text{Product of Roots}) = 0$

$r_1 r_2 = -6$

13. If the length of a rectangular kitchen is half the square of its width and its perimeter is 48 m, find the dimensions of the kitchen.



$P = 48$

$2W + 2L = 48$ ①

$W + L = 24$

$L = \frac{W^2}{2} \Rightarrow 2L = W^2$ ②

② → ①

$2W + W^2 = 48$

$W^2 + 2W - 48 = 0$

$(W - 6)(W + 8) = 0$

$W = 6$ or -8

Doesn't make sense

$\Rightarrow W = 6\text{ m}$

$6 + L = 24$

$L = 18\text{ m}$

15. Find the equation of the quadratic curve that passes through the points $(-2, -1), (1, 2), (3, -16)$.

$f(x) = ax^2 + bx + c$

$f(-2) = a(-2)^2 + b(-2) + c = -1$

$4a - 2b + c = -1$ ①

$f(1) = a(1)^2 + b(1) + c = 2$

$a + b + c = 2$ ②

$f(3) = a(3)^2 + b(3) + c = -16$

$9a + 3b + c = -16$ ③

① - ②
$$4a - 2b + c = -1$$
$$-a - b - c = -2$$
$$3a - 3b = -3$$
$$a - b = -1 \quad ④$$

③ - ②
$$9a + 3b + c = -16$$
$$-a - b - c = -2$$
$$8a + 2b = -18$$
$$4a + b = -9 \quad ⑤$$

⑤ + ④
$$4a + b = -9$$
$$a - b = -1$$
$$5a = -10$$
$$a = -2$$

→⑤ $4(-2) + b = -9$

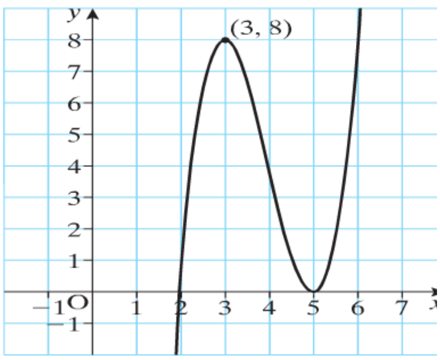
$b = -1$

→② $-2 - 1 + c = 2$

$c = 5$

$f(x) = -2x^2 - x + 5$

17. A section of the graph of a polynomial
$$f(x) = ax^3 + bx^2 + cx + d$$
is drawn in this diagram.
(i) Find the roots of this polynomial.
(ii) Write an expression for $f(x)$ in terms of the factors of this polynomial.
(iii) Find the values of a, b, c and d.
(iv) Find an expression for the reflected image of this curve in the x-axis.
(v) Find an expression for the reflected image of this curve in the y-axis.



(i) Roots = 2 and 5

(ii) $f(x) = k(x-2)(x-5)(x-5)$

$f(3) = k(3-2)(3-5)(3-5) = 8$

$k(1)(-2)(-2) = 8$

$k(4) = 8$

$k = 2$

$\Rightarrow f(x) = 2(x-2)(x-5)(x-5)$

(iii) $f(x) = 2(x-2)(x-5)^2$

$= [2x - 4][x^2 - 10x + 25]$

$= 2x^3 - 20x^2 + 50x - 4x^2 + 40x - 100$

$= 2x^3 - 24x^2 + 90x - 100$

$\Rightarrow a = 2,\ b = -24,\ c = 90,\ d = -100$

Reflected image $g(x) = -f(x)$

$g(x) = -2x^3 + 24x^2 - 90x + 100$