7. In each of the following, express a in terms of the other variables:

(i) $\frac{x}{y} = \frac{a+b}{a-b}$ (ii) $bc - ac = ac.$

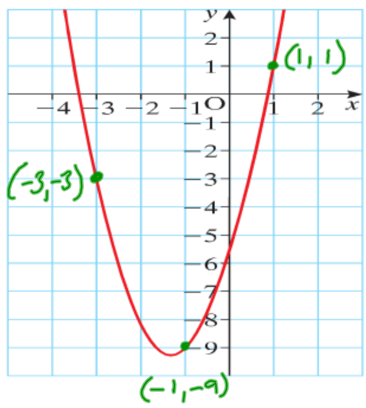
	(i)		(ii)
multiply by LCM	$x(a-b) = y(a+b)$	divide by c	$b - a = a$
expand	$ax - bx = ay + by$	add a to both sides	$b = 2a$
group a's on LHS	$ax - ay = bx + by$	divide by 2	$\frac{b}{2} = a$
factorise	$a(x-y) = b(x+y)$		
divide	$a = \frac{b(x+y)}{(x-y)}$		

8. Express v in terms of the other variables in each of the following:

(i) $y = \frac{3(u-v)}{4}$ (ii) $S = \frac{t}{2}(u+v)$

	(i)		(ii)
Multiply by 4 Divide by 3	$\frac{4y}{3} = u - v$	Multiply by 2 Divide by t	$\frac{2s}{t} = u + v$
Subtract u from both sides	$\frac{4y}{3} - u = -v$	Take u from both sides	$\frac{2s}{t} - u = v$
change all signs	$v = u - \frac{4y}{3}$		

9. A curve of the form $f(x) = y = ax^2 + bx + c$ is drawn as shown.
By picking any three points on the curve, form three equations connecting the coefficients a, b and c and hence solve to find $f(x)$.



pt. $(-3,-3) \Rightarrow a(-3)^2 + b(-3) + c = -3$

① $9a - 3b + c = -3$

pt. $(1,1) \Rightarrow a(1)^2 + b(1) + c = 1$

② $a + b + c = 1$

pt $(-1,-9) \Rightarrow a(-1)^2 + b(-1) + c = -9$

③ $a - b + c = -9$

using ② and ③

$$\begin{array}{r} a + b + c = 1 \\ -a + b - c = 9 \\ \hline 2b \quad = 10 \\ b \quad = 5 \end{array}$$

Sub into ①

$9a - 3(5) + c = -3$

④ $9a + c = 12$

Sub into ②

$a + 5 + c = 1$

⑤ $a + c = -4$

④−⑤ $8a = 16$

$a = 2$

$2 + c = -4$

$c = -6$

Hence Solve $f(x) = ax^2 + bx + c$

$a = 2 \qquad b = 5 \qquad c = -6$

$\Rightarrow f(x) = 2x^2 + 5x - 6$

Solve $\Rightarrow 2x^2 + 5x - 6 = 0$ use formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-5 \pm \sqrt{(5)^2 - 4(2)(-6)}}{2(2)} = \frac{-5 \pm \sqrt{25 + 48}}{4} = \frac{5 \pm \sqrt{73}}{4}$$

$$x \simeq \frac{5 \pm 8.54}{4} \Rightarrow x = \frac{5 + 8.54}{4} = \frac{13.54}{4} = 3.39$$

or $x = \dfrac{5 - 8.54}{4} = \dfrac{-3.54}{4} = 0.89$

10. 44,000 people attended a match in Croke Park. The two ticket prices on the day were €30 and €20. The total receipts for the game came to €1.2 million.
How many people paid the higher ticket price?

let x = no. of people who paid the higher price

let y = no. of people who paid the lower price

$x + y = 44{,}000$

$30x + 20y = 1{,}200{,}000$

$3x + 2y = 120{,}000$

$3x + 2y = 120{,}000$

$-2x - 2y = -88{,}000$

$x = 32{,}000$

$\Rightarrow y = 12{,}000$

7. Given that (any real number)$^2 \geqslant 0$, prove that the following equations have real roots for all values of $k \in R$.

(i) $x^2 - 3kx - k^2 = 0$ (ii) $kx^2 + 2x + (2 - k) = 0$

If Discriminent $= b^2 - 4ac \geq 0$

$\Rightarrow$ Real roots

(i) $a = 1$

$b = -3k$

$c = -k^2$

$\Rightarrow b^2 - 4ac = (-3k)^2 - 4(1)(-k^2)$

$= 9k^2 + 4k^2$

$= 13k^2 \geq 0$

$\Rightarrow$ It has real roots

(ii) $a = k$

$b = 2$

$c = 2 - k$

$\Rightarrow b^2 - 4ac = (2)^2 - 4(k)(2-k)$

$= 4 - 8k + 4k^2$

$= 4k^2 - 8k + 4$

$= (2k - 2)(2k - 2)$

$= (2k-2)^2 \geq 0$

$\Rightarrow$ It has real roots

8. Show that the roots of the equation $x^2 - 3x + 2 - c^2 = 0$ are real for all values of $c \in R$.

If Discriminent $= b^2 - 4ac \geq 0$

$\Rightarrow$ Real roots

$a = 1$

$b = -3$

$c = 2 - c^2$

$$b^2 - 4ac = (-3)^2 - 4(1)(2 - c^2)$$
$$= 9 - 8 + 4c^2$$
$$= 1 + 4c^2 > 0$$

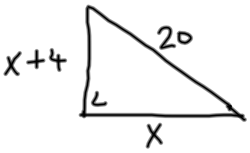
$\Rightarrow$ Real Roots

7. A football is kicked up into the air. The height of the ball can be modelled by the equation $h = -16t^2 + 24t + 1$, where h = the height in metres and t = time in seconds.
At what times will the ball be at a height of 6 m?

$$6 = -16t^2 + 24t + 1$$

$$-16t^2 + 24t - 5 = 0$$
$$16t^2 - 24t + 5 = 0$$
$$(4t - 5)(4t - 1) = 0$$
$$t = \frac{5}{4} \quad \Big| \quad t = \frac{1}{4}$$

8. One side of a right-angled triangle is 4 cm longer than the other side. The hypotenuse is 20 cm long. Find the shortest side of the triangle.

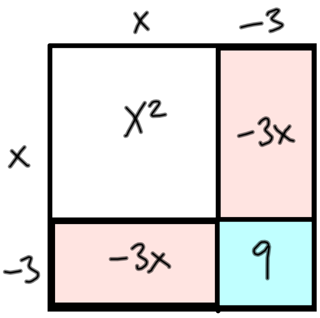


Pythagoras

$$(x+4)^2 + x^2 = 20^2$$
$$x^2 + 8x + 16 + x^2 = 400$$
$$2x^2 + 8x - 384 = 0$$
$$x^2 + 4x - 192 = 0$$
$$(x+16)(x-12) = 0$$
$$x = -16 \quad \text{or} \quad 12$$

5. (i) Complete the square of the expression $x^2 - 6x + k$.
(ii) Find the minimum value of k such that $x^2 - 6x + k$ is positive for all values of x.

(i)



$$x^2 - 6x + 9$$

(ii)

$$x^2 - 6x + k > 0$$
$$x^2 - 6x + 9 + k - 9 > 0$$
$$(x-3)^2 + k - 9 > 0$$

Since $(x-3)^2 \geq 0$

$$\Rightarrow \quad k - 9 > 0$$
$$\Rightarrow \quad k > 9$$

6. Express $2x^2 - 12x + 7$ in the form $a(x - b)^2 + c$.

	x	-3
x	x^2	$-3x$
-3	$-3x$	9

$$2\left[x^2 - 6x + \frac{7}{2}\right]$$

$$= 2\left[(x-3)^2 - 9 + \frac{7}{2}\right]$$

$$= 2(x-3)^2 - 18 + 7$$

$$= 2(x-3)^2 - 11$$