Derive

Amortisation: $A = \frac{P i (1+i)^t}{(1+i)^t - 1}$

Formula

P = loan amount
A = each repayment
t = time units of the loan
i = interest rate

$$\boxed{F = P(1+i)^t}$$

let L = amount still outstanding

after $t=1$

$F_1 = P(1+i)$

we then Repay A

$L_1 = P(1+i) - A$

after $t=2$

$F_2 = L_1(1+i) = [P(1+i) - A](1+i)$

$F_2 = P(1+i)^2 - A(1+i)$

we then repay A

$L_2 = P(1+i)^2 - A(1+i) - A$

after $t=3$

$F_3 = L_2(1+i) = [P(1+i)^2 - A(1+i) - A](1+i)$

$F_3 = P(1+i)^3 - A(1+i)^2 - A(1+i)$

we then repay A

$L_3 = P(1+i)^3 - A(1+i)^2 - A(1+i) - A$

*After time t the loan will be fully repayed

after time $t = t$

$L_t = P(1+i)^t - A(1+i)^{t-1} - \ldots - A(1+i) - A = 0$*

A is HCF, notice geometric series

$$P(1+i)^t - A\underbrace{\left[(1+i)^{t-1} + \ldots + (1+i)^1 + 1\right]}_{\text{Geometric series}} = 0$$

Sum the geometric series

$$\boxed{S_n = \frac{a(1-r^n)}{1-r}}$$

$a = 1$ $\quad r = (1+i)$ $\quad n = t$

$$S_t = \frac{1(1-(1+i)^t)}{1-(1+i)}$$

$$= \frac{1-(1+i)^t}{\cancel{1}-\cancel{1}-i}$$

$$= \frac{(1+i)^t - 1}{i}$$

$$P(1+i)^t - A\left(\frac{(1+i)^t - 1}{i}\right) = 0$$

$$P(1+i)^t = A\left(\frac{(1+i)^t - 1}{i}\right)$$

$$\frac{P i (1+i)^t}{(1+i)^t - 1} = A$$

QED