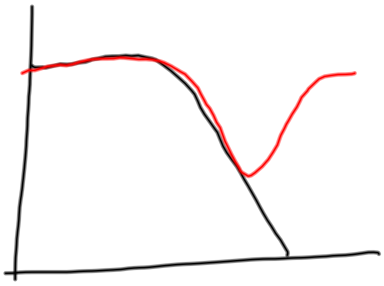
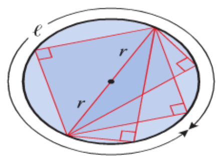
Area and Volume

Section 6.2



Section 6.2 Sectors of circles

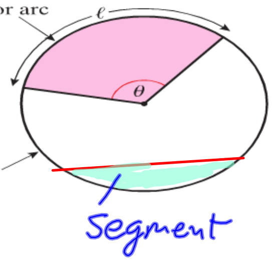
1. Revision of circles and sectors of circles

Circle/Disc		• perimeter (circumference) $= \ell$;s ince $\frac{\ell}{2r} = \pi$ $\Rightarrow \ell = 2\pi r$ • area $= \pi r^2$ • a cyclic quadrilateral is a quadrilateral inscribed in a circle • every triangle inscribed in a semicircle is a right-angled triangle • **360° = 2π radians**

2. Arc of a circle

In the chapter on trigonometry, the length of an arc, the area of a sector, and radian measure were introduced. The length of an arc of a circle is found using the ratios

$$\frac{\ell}{2\pi r} = \frac{\theta\,(\text{degrees})}{360} = \frac{\theta\,(\text{radians})}{2\pi}$$



$$\therefore \text{ Length of arc } (l) = 2\pi r \frac{\theta\,(\text{degrees})}{360} = 2\pi r \frac{\theta\,(\text{radians})}{2\pi} = r\theta \;\; (\theta \text{ in radians})$$

3. Area of a sector

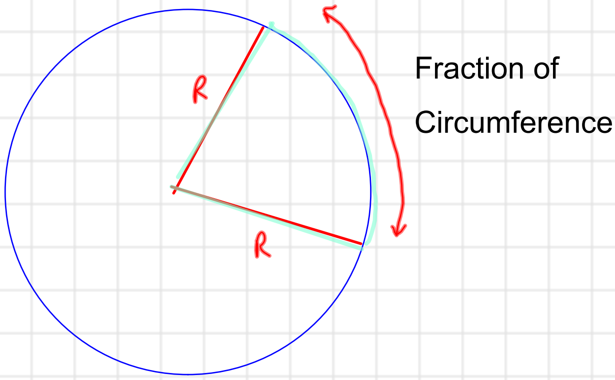
Similarly, the area of a sector of a circle is found using the ratios

$$\frac{A}{\pi r^2} = \frac{\theta\,(\text{degrees})}{360} = \frac{\theta\,(\text{radians})}{2\pi}$$

$$\therefore \text{ Area of sector } (A) = \pi r^2 \frac{\theta\,(\text{degrees})}{360} = \pi r^2 \frac{\theta\,(\text{radians})}{2\pi} = \tfrac{1}{2} r^2 \theta \; (\theta \text{ in radians})$$

Perimeter of Sector

$C = 2\pi R$



Perimeter $= 2r +$ fraction of C

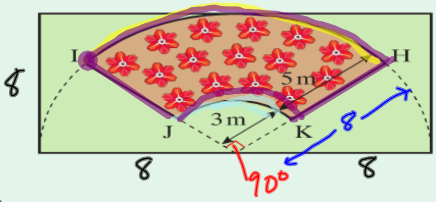
Example 1

A flowerbed in the shape of a section of a sector of a circle is placed in the centre of a rectangular lawn, as shown in the diagram. Calculate

(i) the length of edging needed for the flowerbed

(ii) the area of grass in the garden.

Correct each answer to one place of decimals.



$C = 2\pi R$

(i) $\text{Arc}_{IH} = \frac{1}{4}(2\pi 8) = 4\pi$

$\text{Arc}_{JK} = \frac{1}{4}(2\pi 3) = \frac{3}{2}\pi$

Perimeter flowerbed $P = 5 + 5 + 4\pi + \frac{3}{2}\pi = 10 + \frac{11}{2}\pi \approx 27.3\text{ m}$

(ii) Rectangle $A = LB = 8(16) = 128\text{ m}^2$

Small sector $= \frac{1}{4}(\pi(3)^2) = \frac{9}{4}\pi$

Large sector $= \frac{1}{4}(\pi(8)^2) = 16\pi$

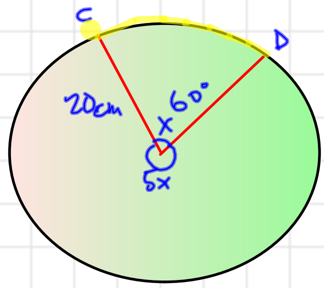
Flower area $= 16\pi - \frac{9}{4}\pi = \frac{55}{4}\pi$

Grass area $= 128 - \frac{55}{4}\pi \approx 84.8\text{ m}^2$

Example 2

A minor arc CD of a circle, centre O and radius 20 cm, subtends an angle x radians at O. The major arc CD of the circle subtends an angle $5x$ radians at O. Find, in terms of π, the length of the minor arc.

$C = 2\pi R$



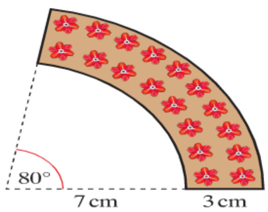
$6x = 360°$

$x = 60°$

$Arc_{CD} = \frac{1}{6} 2\pi(20) = \frac{20}{3}\pi$

Exercise 6.2

1. A drawing of a curved flower bed is shown. The scale in the drawing is 1 cm : 1 m. Calculate, correct to 1 place of decimals,
   (i) the perimeter of the bed
   (ii) the area of the bed.



(i) Perimeter?

Flowerbed perimeter has a small arc + large arc + 2 sides of length 3.

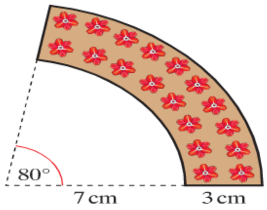
$\text{Arc length} = \frac{\theta}{360°}(2\pi r)$

$$P = \underbrace{\frac{80}{360}(2)(3.14)(10)}_{\text{large arc}} + \underbrace{\frac{80}{360}(2)(3.14)(7)}_{\text{small arc}} + 3 + 3$$

$P = 29.7$ cm

Exercise 6.2

1. A drawing of a curved flower bed is shown.
The scale in the drawing is 1 cm : 1 m.
Calculate, correct to 1 place of decimals,
(i) the perimeter of the bed
(ii) the area of the bed.



(ii) Area?

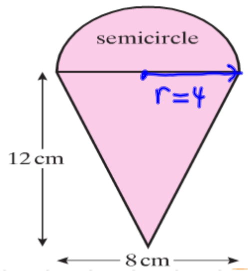
Area of sector $= \frac{\theta}{360}\pi r^2$

Area Small sector $= \frac{80}{360}(3.14)(7)^2 = 34.2 \text{ cm}^2$

Area Large sector $= \frac{80}{360}(3.14)(10)^2 = 69.8 \text{ cm}^2$

Area flowerbed $= 69.8 - 34.2 = 35.6 \text{ cm}^2$

2. Find:
(i) the total area, correct to the nearest cm²
(ii) the total perimeter enclosed by this composite figure, correct to the nearest cm.



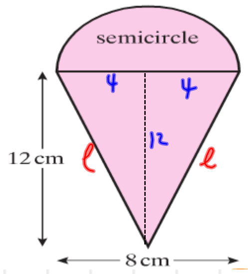
(i) Area

Area Semicircle $= \frac{\pi R^2}{2}$ $= \frac{(3.14)(4)^2}{2} = 25.12 \text{ cm}^2$

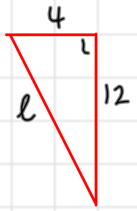
Area triangle $= \frac{Bh}{2}$ $= \frac{12(8)}{2} = 48 \text{ cm}$

Total Area $= 48 + 25.12 \approx 73 \text{ cm}$ (n.w.n)

2. Find:
(i) the total area, correct to the nearest cm^2
(ii) the total perimeter enclosed by this composite figure, correct to the nearest cm.



(ii) Perimeter = arc + 2 sides of triangle (ℓ)



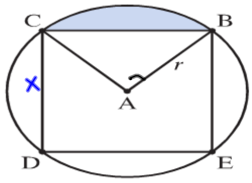
$\ell^2 = 4^2 + 12^2 = 16 + 144$
$\ell = \sqrt{160} = 4\sqrt{10}$

Arc length $= \frac{\theta}{360°}(2\pi r)$

$= \frac{180}{360}(2)(3{\cdot}14)(4) = 12{\cdot}56$

$P = 4\sqrt{10} + 12{\cdot}56 = 25$ cm (n.a.n.)

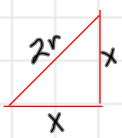
6. A square is inscribed inside a circle of radius r. Find
(i) the area of the square BCDE
(ii) the shaded area in terms of r.



(i)

diagonal of square $= 2r$

$A = LB$



$(2r)^2 = x^2 + x^2$
$4r^2 = 2x^2$
$2r^2 = x^2$
$x = r\sqrt{2}$

$A = LB$

Area Square $= (r\sqrt{2})^2 = 2r^2$

(ii)

Shadded area = Sector − triangle

angle 90° Base = r, height = r

$A_\Delta = \frac{Bh}{2}$

$A = \frac{\pi r^2}{4} - \frac{r^2}{2}$

$A_{sector} = \frac{\theta}{360}\pi r^2$