4. g, f, c method

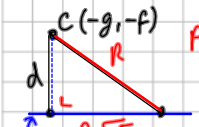
e.g. find the equation of the circle which contains the points $(7,5)$ and $(8,-2)$ if the line $2x-y-2=0$ contains a chord of length $4\sqrt{5}$ of the circle

$x^2+y^2+2gx+2fy+c=0$

we want 3 equations!

Sub in points { contains $(7,5)\Rightarrow$; contains $(8,-2)\Rightarrow$

Since $\perp$ line from centre to chord bisects chord

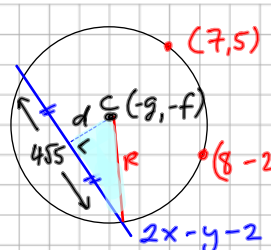


Pythagoras: $d=\sqrt{R^2-20}$

$R=\sqrt{g^2+f^2-c}$

also $d=\dfrac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$

Sketch



$(7)^2+(5)^2+2g(7)+2f(5)+c=0$

$49+25+14g+10f+c=0$

$14g+10f+c=-74$ (1)

$(8)^2+(-2)^2+2g(8)+2f(-2)+c=0$

$64+4+16g-4f+c=0$

$16g-4f+c=-68$ (2)

$\Rightarrow \sqrt{(\sqrt{g^2+f^2-c})^2-20}=\dfrac{|2(-g)-1(-f)-2|}{\sqrt{2^2+1^2}}$

$g^2+f^2-c-20=\left(\dfrac{-2g+f-2}{5}\right)^2$

$5g^2+5f^2-5c-100=4g^2-2gf+4g$

$-2gf+f^2-2f+4g-2f+4$

$g^2+4f^2+4gf+4f-8g-5c-104=0$ (3)

Solve 3 equations

(1) $\rightarrow$ $c=-16g+4f-68$

(2) - (1)

$2g-14f=6$

$g-7f=3$, $g=7f+3$ (4)

(4) $\rightarrow$ (1)

$c=-16(7f+3)+4f-68$

$=-112f-48+4f-68$

$c=-108f-116$ (5)

(4) & (5) $\rightarrow$ (3)

$14g+10f+c=-74$ (1)

$16g-4f+c=-68$ (2)

$g^2+4f^2+4gf+4f-4g-5c-104=0$ (3)

$(7f+3)^2+4f^2+4(7f+3)f+4f$

$-8(7f+3)-5(108f-116)-104=0$

Solve quadratic

$49f^2+42f+9+4f^2+28f^2+12f$

$+4f-56f-24+540f+580-104=0$

$81f^2+542f+461=0$

$(81f+461)(f+1)$

$f=-\dfrac{461}{81}$ or $f=-1$

$g=7(-1)+3=-7+3=-4$

$g=-4$

$c=-108(-1)-116=-8$

$c=-8$

Centre $=(-g,-f)$

Centre $=(4,1)$

Radius $=\sqrt{g^2+f^2-c}$

Radius $=\sqrt{4^2+1^2-8}=3$

Circle equation:

$x^2+y^2+2gx+2fy+c=0$

$x^2+y^2-8x-2y-8=0$