Equality of complex numbers

For two complex numbers to be equal, their real parts must be equal and their imaginary parts must be equal.

If $(x + 2) + 4i = 6 + (y - 2)i$,
then $x + 2 = 6$ and $4 = y - 2$
$\Rightarrow x = 4$ and $6 = y$

If $a + bi = x + yi$,
then $a = x$ and $b = y$

In a Complex Equation

- Real parts = Real parts
- Imaginary parts = Imaginary parts

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Example 2

Find x and y if $x + 2i + 2(3 - 5yi) = 8 - 13i$.

$$x + 2i + 2(3 - 5yi) = 8 - 13i$$
$$\Rightarrow \quad x + 2i + 6 - 10yi = 8 - 13i$$
$$\Rightarrow \quad x + 6 + (2 - 10y)i = 8 - 13i$$

Equating the real parts:

$$x + 6 = 8$$
$$x = 2$$

Equating the imaginary parts:

$$2 - 10y = -13$$
$$-10y = -15$$
$$10y = 15$$
$$y = \tfrac{15}{10} = \tfrac{3}{2}$$

6. Find the values of x and y in each of the following:

(i) $x + yi = 4 - 2i$ (ii) $x + yi = (2 + i)(3 - 2i)$

(iii) $x + yi = \dfrac{7 + i}{2 - i}$ (iv) $x + yi = (2 - 3i)^2$

(i) $x = 4$

$y = -2$

(ii) $x + yi = 6 - 4i + 3i \overset{+}{-} 2\cancel{i^2}$

$= 8 - i$

$\Rightarrow x = 8$

$y = -1$

(iii) $x + yi = \dfrac{(7+i)(2+i)}{(2-i)(2+i)}$

$= 14 + 7i + 2i \overset{-}{+} 1\cancel{i^2}$

$= 13 + 9i$

$\Rightarrow x = 13$

$y = 9$

(iv) $x + yi = (2 - 3i)^2$

$= 4 - 12i \overset{-}{+} 9\cancel{i^2}$

$= -5 - 12i$

$\Rightarrow x = -5$

$y = -12$

7. Find the values of a and b in each of the following:

(i) $a + bi + 3 - 2i = 4(-2 + 5i)$

(ii) $a(1 + 2i) - b(3 + 4i) = 5$

(i) $a + bi + 3 - 2i = -8 + 20i$

$a + bi = -11 + 22i$

$\Rightarrow a = 11 \quad , \quad b = 22$

(ii) $a + 2ai - 3b - 4bi = 5 + 0i$

Re = Re

$\Rightarrow a - 3b = 5$

Im = Im

$2a - 4b = 0$

$a = 2b$

$\Rightarrow 2b - 3b = 5$

$-b = 5$

$b = -5$

$a = 2(5) = -10$

8. If $z = x + yi$ and $3(z - 1) = i(3 + i)$, find the values of x and y.

$$3z - 1 = 3i \mp 1\cancel{i^2}$$

$$3z - 1 = 3i - 1$$

$$\Rightarrow \quad z = i$$

$$\Rightarrow \quad x + yi = 0 + 1i$$

$$\Rightarrow \quad x = 0$$

$$y = 1$$