

4 If  $z = 2 + 6i$ , express each of the following in the form of  $a + bi$ , where  $a, b \in \mathbb{R}$ :

(i)  $z \cdot \bar{z}$

(ii)  $z + \bar{z}$

(iii)  $z - \bar{z}$

(iv)  $z^2$

$$\bar{z} = 2 - 6i$$

$$(i) z \cdot \bar{z} = (2 + 6i)(2 - 6i) = 4 - 36i^2 = 40 \quad \checkmark$$

Difference of 2 Squares

$$(ii) z + \bar{z} = 2 + 6i + 2 - 6i = 4 \quad \checkmark$$

$$(iii) z - \bar{z} = 2 + 6i - (2 - 6i) = 2 + 6i - 2 + 6i = 12i \quad \checkmark$$

$$(iv) z^2 = (2 + 6i)^2 = (2 + 6i)(2 + 6i)$$

$$= 2(2 + 6i) + 6i(2 + 6i)$$

$$= 4 + 12i + 12i + 36i^2$$

$$= -32 + 24i \quad \checkmark$$

5. Simplify each of the following.

(i)  $\frac{(3 + 4i) + (2 + i)}{4 - i}$

(ii)  $\frac{(2 - 6i) - (3 + 2i)}{2 + 2i}$

Simplify the numerator

multiply above and below by  
conjugate of the denominator

expand

$$(i) = \frac{5 + 5i}{4 - i} = \frac{(5 + 5i)(4 + i)}{(4 - i)(4 + i)} = \frac{20 + 5i + 20i + 5i^2}{16 - i^2}$$

Difference of 2 Squares

$$= \frac{15 + 25i}{17} = \frac{15}{17} + \frac{25i}{17} \quad \checkmark$$

$$(ii) = \frac{2 - 6i - 3 - 2i}{2 + 2i} = \frac{-1 - 8i}{2 + 2i} = \frac{(-1 - 8i)(2 - 2i)}{(2 + 2i)(2 - 2i)}$$

$$= \frac{-2 + 2i - 16i + 16i^2}{4 - 4i^2} = \frac{-18 - 14i}{8} = \frac{-9}{4} - \frac{7}{4}i \quad \checkmark$$

$$(iii) \frac{3(2+4i)}{5i}$$

$$= \frac{6+12i}{5i}$$

$$= \frac{(6+12i)(-5i)}{(5i)(-5i)}$$

$$= \frac{-30i + 60i^2}{+25i^2} = \frac{60-30i}{25}$$

$$= \frac{12-6i}{5}$$

$$(iv) \frac{(2+i) + (3-2i)}{(4+i) - (3+2i)}$$

$$= \frac{5+3i}{1-i}$$

$$= \frac{(5+3i)(1+i)}{(1-i)(1-i)}$$

$$= \frac{5+5i+3i+3i^2}{1+i^2}$$

$$= \frac{2+8i}{2} = 1+4i \quad ?$$

$$(v) \frac{(3+2i)(1-i)}{2+4i}$$

$$= \frac{3-3i+2i+2i^2}{2+4i}$$

$$= \frac{5-i}{2+4i}$$

$$= \frac{(5-i)(2-4i)}{(2+4i)(2-4i)}$$

$$= \frac{10-20i-2i+4i^2}{4+16i^2}$$

$$= \frac{6-22i}{20} = \frac{3}{5} - \frac{11}{10}i \quad \checkmark$$

$$(vi) \frac{(3+i)(2-i)}{(4+i)(2+i)}$$

$$= \frac{6-3i+2i+i^2}{8+4i+2i+i^2}$$

$$= \frac{7-i}{7+6i}$$

$$= \frac{(7-i)(7-6i)}{(7+6i)(7-6i)}$$

$$= \frac{49-42i-7i+6i^2}{49+36i^2}$$

$$= \frac{43-49i}{85} = \frac{43}{85} - \frac{49}{85}i \quad \checkmark$$