

- II
6. Given that $2+3i$ is one root of the equation $2z^3 - 9z^2 + 30z - 13 = 0$, find the other two roots.

Solve CUBIC

- ① Conjugate is 2nd Root
- ② Use complex Roots to write quadratic factor
- ③ Divide to get linear factor
- ④ Write Roots

 $2-3i$ is also a root

$$z^2 - (R_1+R_2)z + (R_1R_2) = 0$$

$$R_1+R_2 = 2+3i + 2-3i = 4$$

$$R_1R_2 = (2-3i)(2+3i) = (4-9i^2)$$

Diff-2 SQUARES

$$z^2 - (4)z + (4+9) = 0$$

$$z^2 - 4z + 13 = 0$$

Dividing

factor

$$\begin{array}{r} 2z-1 \\ \hline z^2 - 4z + 13 \end{array}$$

$$\begin{array}{r} 2z^2 - 9z^2 + 30z - 13 \\ \cancel{+ 2z^3} \cancel{+ 8z^2} \cancel{- 26z} \\ -z^2 + 4z - 13 \\ \cancel{+ z^2} \cancel{+ 4z} \cancel{- 13} \end{array}$$

$$\Rightarrow 2z-1=0$$

$$2z=1$$

$$z = \sqrt{2}$$

3 roots are

$$z = 2-3i, 2+3i, \frac{1}{2}$$

- II
8. $\frac{1+2i}{1-2i}$ is a root of $az^2 + bz + 5 = 0$, where $a, b \in R$. Find a value for a and for b .

- ① Divide - Rationalise denominator
- ② Conjugate is other root
- ③ Use roots to form equation
- ④ Compare to show values for a and b

$$\frac{(1+2i)(1+2i)}{(1-2i)(1+2i)} = \frac{1+4i+4i^2}{1-4i^2} = \frac{-3+4i}{5}$$

$$= -\frac{3}{5} + \frac{4}{5}i \quad \text{other root} \quad -\frac{3}{5} - \frac{4}{5}i$$

$$z^2 - (R_1+R_2)z + (R_1R_2) = 0$$

$$R_1+R_2 = -\frac{3}{5} - \frac{3}{5} = -\frac{6}{5}$$

$$R_1R_2 = \left(\frac{9}{25}\right) \pm \left(\frac{16}{25}\right)i^2 = \frac{25}{25} = 1$$

$$\Rightarrow z^2 + \frac{6}{5}z + 1 = 0$$

$$5z^2 + 6z + 5 = 0$$

$$\Rightarrow a=5 \quad \text{and} \quad b=6 \quad \text{smiley face}$$

11. Form the quadratic equation whose roots are $-3 \pm 2i$.

Hence form the cubic equation whose roots are $-3 \pm 2i$ and 2.

① use roots
to form
quadratic

$$z^2 - (R_1 + R_2)z + (R_1 R_2) = 0$$

$$R_1 + R_2 = -3 + 2i + -3 - 2i = -6$$

$$R_1 R_2 = (-3+2i)(-3-2i) = 9 + 4i^2 = 13$$

$$\Rightarrow z^2 + 6z + 13 = 0$$

② Write other
factor from
Root

If $z = 2$ is a solution $\Rightarrow (z-2)$ is factor

③ multiply
quadratic & linear
factors to get
cubic

$$\begin{aligned} (z-2)(z^2 + 6z + 13) &= 0 \\ &= z(z^2 + 6z + 13) - 2(z^2 + 6z + 13) \\ &= z^3 + 6z^2 + 13z - 2z^2 - 12z - 26 \end{aligned}$$

$$\Rightarrow z^3 + 4z^2 + z - 26 = 0 \quad \text{😊}$$

notes

The Polar form of a

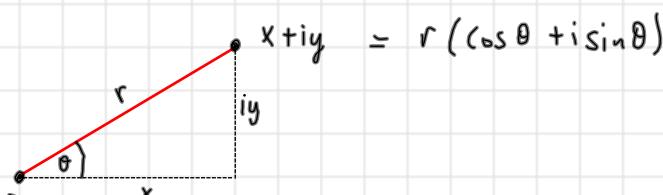
Complex Number

Rectangular form / Cartesian form: $x + iy$

Polar form / Modulus argument form: $r(\cos \theta + i \sin \theta)$

r = modulus

θ = argument



this is true because

$$\sin \theta = \frac{iy}{r} \Rightarrow iy = r \sin \theta$$

$$\cos \theta = \frac{x}{r} \Rightarrow x = r \cos \theta$$

ANGLES

Degrees

Converting:

$$360^\circ \leftrightarrow 2\pi$$

$$180^\circ \leftrightarrow \pi$$

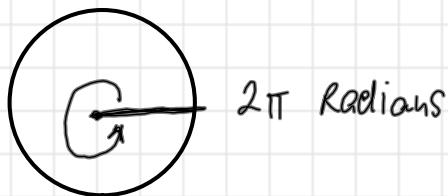
$$45^\circ \leftrightarrow \pi/4$$

$$90^\circ \leftrightarrow \pi/2$$

$$60^\circ \leftrightarrow \pi/3$$



RADIANs



Example 1

Express in the form $x + iy$ these complex numbers:

(a) $z_1 = 12 \left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right)$

(b) $z_2 = 5 \left(\cos \frac{\pi}{8} + i \sin \frac{\pi}{8} \right)$

Note: When angle is written in terms of π the angle is in Radians!

use the calculator to evaluate

$$180^\circ = \pi \text{ Radian}$$

$$30^\circ = \pi/6 \text{ Rads}$$

convert
Radians \rightarrow degrees

$$\begin{aligned} z_1 &= 12 \left(\cos 30^\circ + i \sin 30^\circ \right) \\ &= 12 \left(\frac{\sqrt{3}}{2} + \frac{1}{2} i \right) \\ &= 6\sqrt{3} + 6i \end{aligned}$$