4. Simplify each of the following, giving your answer in the form $r(\cos\theta + i\sin\theta)$.

(i) $(1 + i\sqrt{3})^2$ (ii) $\dfrac{-2}{-\sqrt{3} + i}$

3.7

DIVIDE

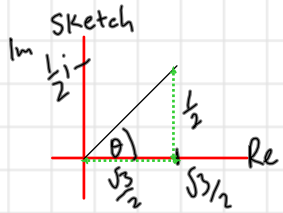
$$\frac{(-2)(-\sqrt{3}-i)}{(-\sqrt{3}+i)(-\sqrt{3}-i)} = \frac{2\sqrt{3}+2i}{3 \pm 1\cancel{i^2}} = \frac{2\sqrt{3}+2i}{4} = \frac{\sqrt{3}}{2} + \frac{1}{2}i$$

modulus, r

$$r = \sqrt{\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = \sqrt{\frac{3}{4}+\frac{1}{4}} = 1$$

argument, θ

Sketch



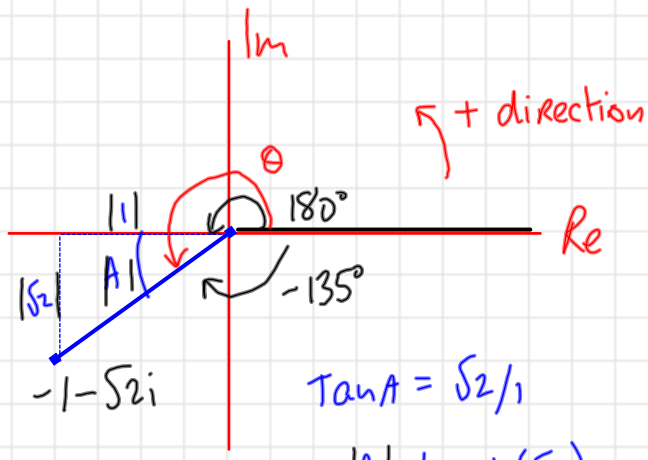
$$\tan\theta = \frac{\left(\frac{1}{2}\right)}{\left(\frac{\sqrt{3}}{2}\right)} = \left(\frac{1}{\cancel{2}}\right)\left(\frac{\cancel{2}}{\sqrt{3}}\right) = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{3}}{3}\right) = 30^\circ$$

ANS: $= 1(\cos 30^\circ + i\sin 30^\circ)$

What is the argument of this complex number?

$\theta = ?$



$\tan A = \sqrt{2}/1$

$|A| = \tan^{-1}(\sqrt{2})$

$\approx 55^\circ$

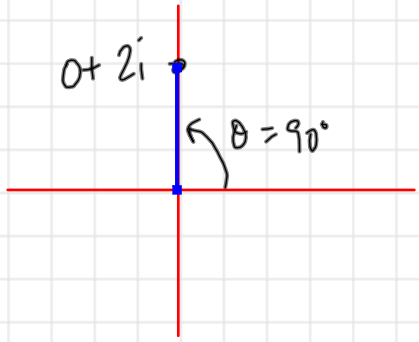
$\theta = 55^\circ + 180^\circ = 235^\circ$

3.7 Q6

p.121

Write in Polar form?

$0 + 2i$



$r = \sqrt{0^2 + 2^2} = 2$

$\theta = 90°$

$2i = 2(\cos 90° + i \sin 90°)$ ✓

3.8

multiply

and

divide

Polars

If $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$

and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$,

then $z_1 . z_2 = r_1 . r_2[\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)]$

and $\frac{z_1}{z_2} = \frac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)]$.

Ex 3.8

If $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$
and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$,
then $z_1.z_2 = r_1.r_2[\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)]$
and $\frac{z_1}{z_2} = \frac{r_1}{r_2}[\cos(\theta_1-\theta_2) + i\sin(\theta_1-\theta_2)]$.

Example 1

If $z_1 = 2\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$ and $z_2 = 5\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right)$, find

(i) z_1z_2 (ii) $\frac{z_2}{z_1}$ in the form $a + ib$.

$$z_1z_2 = (2)(5)\left[\cos\left(\frac{\pi}{4}+\frac{5\pi}{12}\right) + i\sin\left(\frac{\pi}{4}+\frac{5\pi}{12}\right)\right]$$

$$= 10\left[\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right]$$

$$= 10\left[\cos 120^\circ + i\sin 120^\circ\right]$$

$$= 10\left[-\tfrac{1}{2} + i\tfrac{\sqrt{3}}{2}\right]$$

$$= -5 + 5\sqrt{3}\,i$$

RAD $\longrightarrow$ DEG

$\frac{2\pi}{3} = \frac{2(180)}{3} = 120^\circ$

Calculator

$$\frac{\pi}{4} + \frac{5\pi}{12}$$

$$\frac{3\pi + 5\pi}{12} = \frac{8\pi}{12} = \frac{2\pi}{3}$$

$$\frac{1}{4} + \frac{5}{12} = \frac{2}{3}$$