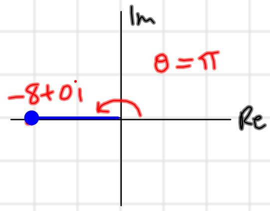
3.10
Applications of
deMoivre's Theorem

5. Find the values of z for which $z^3 = -8$, giving your answer in $a + bi$ form.

(1) write in polar form
$r = ?$ $\theta = ?$

$$z = (-8)^{\frac{1}{3}}$$

$$r = |-8| = 8$$



$$z = \left[8(\cos\pi + i\sin\pi)\right]^{\frac{1}{3}}$$

(2) write in general polar form

$$z = 8^{\frac{1}{3}}\left[\cos(\pi + 2n\pi) + i\sin(\pi + 2n\pi)\right]^{\frac{1}{3}}$$

(3) use de Moivre's theorem

$$z = 2\left[\cos\tfrac{1}{3}(\pi + 2n\pi) + i\sin\tfrac{1}{3}(\pi + 2n\pi)\right]$$

(4) generate results for n=0, n=l, n=2
3 results because a cubic has 3 Solutions

$n = 0$: $z = 2\left[\cos\tfrac{1}{3}(\pi + 2(0)\pi) + i\sin\tfrac{1}{3}(\pi + 2(0)\pi)\right]$
$= 2\left(\tfrac{1}{2} + i\,\tfrac{\sqrt{3}}{2}\right) = 1 + i\sqrt{3}$ ✓

$n = 1$: $z = 2\left[\cos\tfrac{1}{3}(\pi + 2(1)\pi) + i\sin\tfrac{1}{3}(\pi + 2(1)\pi)\right]$
$= 2(-1 + 0i) = -2 + 0i$ ✓

$n = 2$: $z = 2\left[\cos\tfrac{1}{3}(\pi + 2(2)\pi) + i\sin\tfrac{1}{3}(\pi + 2(2)\pi)\right]$
$= 2\left(\tfrac{1}{2} - i\,\tfrac{\sqrt{3}}{2}\right) = 1 - i\sqrt{3}$ ✓

Solutions $z = 1 + i\sqrt{3},\ -2 + 0i,\ 1 - i\sqrt{3}$

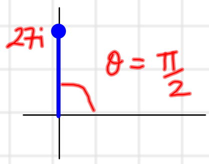
3.10
Applications of
deMoivre's Theorem

8. Find the cube roots of $27i$.

(1) write in polar form
$r = ?$ $\theta = ?$

let $z = \sqrt[3]{27i} = (27i)^{\frac{1}{3}}$

$$r = |27i| = 27$$



$$z = \left[27\left(\cos\tfrac{\pi}{2} + i\sin\tfrac{\pi}{2}\right)\right]^{\frac{1}{3}}$$

(2) write in general polar form

$$z = 27^{\frac{1}{3}}\left[\cos\left(\tfrac{\pi}{2} + 2n\pi\right) + i\sin\left(\tfrac{\pi}{2} + 2n\pi\right)\right]^{\frac{1}{3}}$$

(3) use de Moivre's theorem

$$z = 3\left[\cos\tfrac{1}{3}\left(\tfrac{\pi}{2} + 2n\pi\right) + i\sin\tfrac{1}{3}\left(\tfrac{\pi}{2} + 2n\pi\right)\right]$$

(4) generate results for n=0, n=l, n=2
3 results because a cubic has 3 Solutions

$n = 0$: $z = 3\left[\cos\tfrac{1}{3}\left(\tfrac{\pi}{2} + 2(0)\pi\right) + i\sin\tfrac{1}{3}\left(\tfrac{\pi}{2} + 2(0)\pi\right)\right]$
$= 3\left(\tfrac{\sqrt{3}}{2} + i\tfrac{1}{2}\right) = \tfrac{3\sqrt{3}}{2} + \tfrac{3}{2}i$ ✓

$n = 1$: $z = 3\left[\cos\tfrac{1}{3}\left(\tfrac{\pi}{2} + 2(1)\pi\right) + i\sin\tfrac{1}{3}\left(\tfrac{\pi}{2} + 2(1)\pi\right)\right]$
$= 3\left(-\tfrac{\sqrt{3}}{2} + i\tfrac{1}{2}\right) = -\tfrac{3\sqrt{3}}{2} + \tfrac{3}{2}i$ ✓

$n = 2$: $z = 3\left[\cos\tfrac{1}{3}\left(\tfrac{\pi}{2} + 2(2)\pi\right) + i\sin\tfrac{1}{3}\left(\tfrac{\pi}{2} + 2(2)\pi\right)\right]$
$= 3(0 - 1i) = -3i$ ✓

Solutions **8.** $-3i,\ \frac{3\sqrt{3}}{2} + \frac{3}{2}i,\ -\frac{3\sqrt{3}}{2} + \frac{3}{2}i$