

Sec. 3.10

Applications of deMoivre's theorem

DeMoivre:

$$(\cos \theta + i \sin \theta)^n \\ = \cos(n\theta) + i \sin(n\theta)$$

How can we apply deMoivre's theorem to questions of the form:

$$(\cos \theta - i \sin \theta)^n \quad ?$$

NOTICE: $\cos \theta = \cos(-\theta)$
and $-\sin \theta = \sin(-\theta)$

$$\Rightarrow \cos \theta - i \sin \theta = \cos(-\theta) + i \sin(-\theta)$$

$$(\cos \theta - i \sin \theta)^n = \cos(-n\theta) + i \sin(-n\theta)$$

Exercise 3.10

1. Simplify each of the following, giving your answer in the form $a + bi$:

(i) $(\cos \pi - i \sin \pi)^5$

(ii) $(\cos \frac{\pi}{5} - i \sin \frac{\pi}{5})^{10}$

(iii) $\frac{1}{(\cos \frac{\pi}{3} - i \sin \frac{\pi}{3})^3}$

(iv) $(\cos \frac{\pi}{2} - i \sin \frac{\pi}{2})^4$

$$\begin{aligned} \text{(i)} \quad (\cos \pi - i \sin \pi)^5 &= (\cos(-\pi) + i \sin(-\pi))^5 \\ &= \cos(-5\pi) + i \sin(-5\pi) \\ &= -1 + i0 \end{aligned}$$

use calculator

$$\begin{aligned} \text{(ii)} \quad (\cos \frac{\pi}{5} - i \sin \frac{\pi}{5})^{10} &= [\cos(-\frac{\pi}{5}) + i \sin(-\frac{\pi}{5})]^{10} \\ &= \cos(-\frac{10\pi}{5}) + i \sin(-\frac{10\pi}{5}) \\ &= \cos(-2\pi) + i \sin(-2\pi) \\ &= 1 + 0i \end{aligned}$$

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1. Simplify each of the following, giving your answer in the form $a + bi$:

(i) $(\cos \pi - i \sin \pi)^5$

(ii) $(\cos \frac{\pi}{5} - i \sin \frac{\pi}{5})^{10}$

(iii) $\frac{1}{(\cos \frac{\pi}{3} - i \sin \frac{\pi}{3})^3}$

(iv) $(\cos \frac{\pi}{2} - i \sin \frac{\pi}{2})^4$

$$\begin{aligned} \text{(iii)} \quad \frac{1}{(\cos \frac{\pi}{3} - i \sin \frac{\pi}{3})^3} &= (\cos \frac{\pi}{3} - i \sin \frac{\pi}{3})^{-3} \\ &= [\cos(-\frac{\pi}{3}) + i \sin(-\frac{\pi}{3})]^{-3} \\ &= \cos(-\frac{3\pi}{3}) + i \sin(-\frac{3\pi}{3}) \\ &= \cos(-\pi) + i \sin(-\pi) \\ &= -1 + 0i \end{aligned}$$

$$\begin{aligned} \text{(iv)} \quad (\cos \frac{\pi}{2} - i \sin \frac{\pi}{2})^4 &= [\cos(-\frac{\pi}{2}) + i \sin(-\frac{\pi}{2})]^4 \\ &= \cos(-\frac{4\pi}{2}) + i \sin(-\frac{4\pi}{2}) = \cos(-2\pi) + i \sin(-2\pi) \\ &= 1 + 0i \end{aligned}$$