12 Complex Number Revision Questions

LC HL Project Maths

Complex Number Revision

1. Equality of complex numbers

e.g. find the complex number $z = x + yi$ if

$$4z + 3i\overline{z} = 11 + 10i,$$

where $\overline{z}$ is the conjugate of z

2. Addition, subtraction and multiplication

e.g. if $z = 3 - 2i$ and $w = -3 + 7i$, express in the form $a + ib$:

$$w(2\overline{z} - w)$$

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3. Conjugate and division

e.g. express $\frac{10+5i}{4-3i}$ in the form $a+bi$

4. Square roots

e.g. find the real numbers a and b if

$(a+bi)^2 = 5-12i$

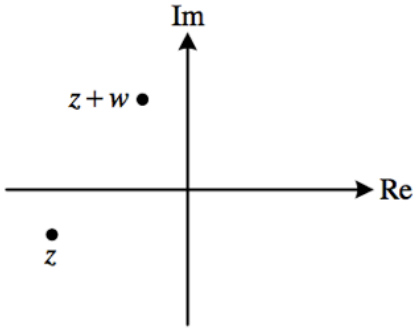
5. Argand diagram and modulus

e.g. $z = 3 + 4i$ and $w = 1 - 2i$. Plot $z + w$ on an Argand diagram and investigate if

$$|z + w| = |z| + |w|$$

6. Interpreting an Argand diagram

e.g. The Argand diagram below shows points representing the complex numbers z and $z + w$.



Copy the diagram and show, with construction, how to locate the points representing the complex numbers

(i) w

(ii) $z - w$

(iii) $2z - w$

(iv) u, if $2u + w = z$.

7. Complex equations

e.g. if $3+i$ is a root of the equation

$$z^2-(a+2i)z+(b+i)=0,$$

find the values of $a, b \in \mathbb{R}$, and find the other root of this equation

8. Conjugate Roots Theorem

e.g. if $1-2i$ is a root of the equation

$$z^3+az^2+bz-40=0, \quad a, b \in \mathbb{R},$$

find the values of a and b and the other roots of the equation

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9. Polar Form

e.g. express $-1-\sqrt{3}i$ in the form

$r(\cos\alpha + i\sin\alpha)$

10. De Moivre: Trigonometric identities

e.g. use De Moivre's Theorem to express $\cos 3\theta$ as a polynomial in $\sin\theta$

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11. De Moivre: Large powers

e.g. express $\left(2\sqrt{3}-2i\right)^6$ in the form $a+bi$

12. De Moivre: Roots

e.g. express the solutions of the equation

$z^4=-81$

in the form $a+bi$. Show these solutions on an Argand diagram.