Sec. 3.10

Demoivre:

$(\cos\theta + i\sin\theta)^n$
$= \cos(n\theta) + i\sin(n\theta)$

Applications of deMoivre's theorem

How can we apply deMoivre's theorem to questions of the form:

$$(\cos\theta - i\sin\theta)^n \quad ?$$

NOTICE: $\cos\theta = \cos(-\theta)$
and $-\sin\theta = \sin(-\theta)$

$$\Rightarrow \cos\theta - i\sin\theta = \cos(-\theta) + i\sin(-\theta)$$
$$(\cos\theta - i\sin\theta)^n = \cos(-n\theta) + i\sin(-n\theta)$$

Exercise 3.10

1. Simplify each of the following, giving your answer in the form $a + bi$:

(i) $(\cos\pi - i\sin\pi)^5$ (ii) $\left(\cos\frac{\pi}{5} - i\sin\frac{\pi}{5}\right)^{10}$

(iii) $\dfrac{1}{\left(\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}\right)^3}$ (iv) $\left(\cos\frac{\pi}{2} - i\sin\frac{\pi}{2}\right)^4$

(i) $(\cos\pi - i\sin\pi)^5 = (\cos(-\pi) + i\sin(-\pi))^5$
$= \cos(-5\pi) + i\sin(-5\pi)$

use calculator $= -1 + i0$

(ii) $\left(\cos\frac{\pi}{5} - i\sin\frac{\pi}{5}\right)^{10} = \left[\cos\left(-\frac{\pi}{5}\right) + i\sin\left(-\frac{\pi}{5}\right)\right]^{10}$
$= \cos\left(-\frac{10\pi}{5}\right) + i\sin\left(-\frac{10\pi}{5}\right)$
$= \cos(-2\pi) + i\sin(-2\pi)$
$= 1 + 0i$

Exercise 3.10

1. Simplify each of the following, giving your answer in the form $a + bi$:

(i) $(\cos \pi - i \sin \pi)^5$ (ii) $\left(\cos \frac{\pi}{5} - i \sin \frac{\pi}{5}\right)^{10}$

(iii) $\dfrac{1}{\left(\cos \frac{\pi}{3} - i \sin \frac{\pi}{3}\right)^3}$ (iv) $\left(\cos \frac{\pi}{2} - i \sin \frac{\pi}{2}\right)^4$

(iii) $\dfrac{1}{(\cos \pi/3 - i\sin \pi/3)^3} = \left(\cos \frac{\pi}{3} - i\sin\frac{\pi}{3}\right)^{-3}$

$= \left[\cos(-\pi/3) + i\sin\left(-\frac{\pi}{3}\right)\right]^{-3}$

$= \cos\left(+\frac{3\pi}{3}\right) + i\sin\left(+\frac{3\pi}{3}\right)$

$= \cos(+\pi) + i\sin(+\pi)$

$= -1 + 0i$

(iv) $\left(\cos\frac{\pi}{2} - i\sin\frac{\pi}{2}\right)^4 = \left[\cos\left(-\frac{\pi}{2}\right) + i\sin\left(-\frac{\pi}{2}\right)\right]^4$

$= \cos\left(-\frac{4\pi}{2}\right) + i\sin\left(-\frac{4\pi}{2}\right) = \cos(-2\pi) + i\sin(-2\pi)$

$= 1 + 0i$

Reciprocals

NB

$\dfrac{1}{3^1} = 3^{-1}$

$\dfrac{1}{3^2} = 3^{-2}$

$\dfrac{2}{x^5} = 2\left(\dfrac{1}{x^5}\right) = 2x^{-5}$

$\left(\dfrac{1}{2}\right)^{-3} = \left(\dfrac{2}{1}\right)^3 = 2^3$

3.10

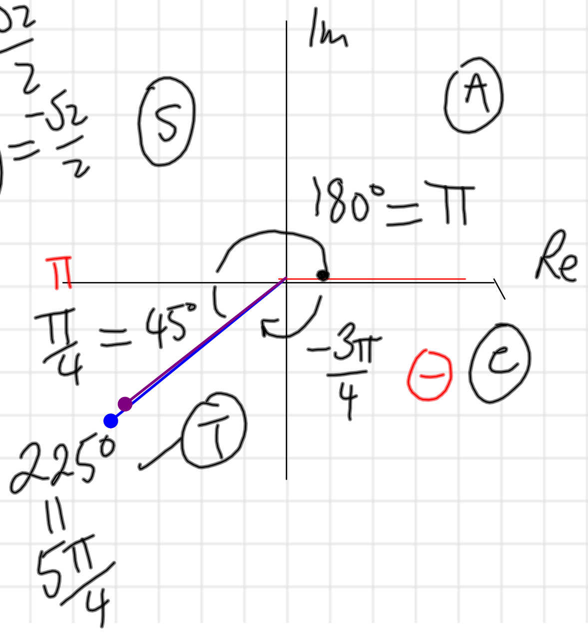
$\cos\frac{5\pi}{4} = -\frac{\sqrt{2}}{2}$

$\cos\left(2\pi + \frac{5\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

$\cos\left(20\pi + \frac{5\pi}{4}\right) = -\frac{\sqrt{2}}{2}$ S

Given $z = a + bi$,
then $z = r[\cos(\theta + 2n\pi) + i\sin(\theta + 2n\pi)]$
where $n \in N$ is the **general polar form** of z.

$\cos\theta = \cos(\theta + 2n\pi)$



3.10
Applications of
deMoivre's
Theorem

note: a cubic has 3 solutions

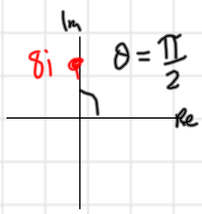
Example 3

Solve the equation $z^3 = 8i$.

1. write in polar form
$r = ?\quad \theta = ?$

$\Rightarrow z = (8i)^{\frac{1}{3}}$

$r = \sqrt{0^2 + 8^2} = 8$



$z = \left[8\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)\right]^{\frac{1}{3}}$

2. write in general polar form

$z = 8^{\frac{1}{3}}\left[\cos\left(\frac{\pi}{2} + 2n\pi\right) + i\sin\left(\frac{\pi}{2} + 2n\pi\right)\right]^{\frac{1}{3}}$

3. use de Moivre's theorem

$z = 2\left[\cos\frac{1}{3}\left(\frac{\pi}{2} + 2n\pi\right) + i\sin\frac{1}{3}\left(\frac{\pi}{2} + 2n\pi\right)\right]$

4. generate results for n=0, n=l, n=2

3 results because a cubic has 3 Solutions

$n=0$: $z = 2\left[\cos\frac{1}{3}\left(\frac{\pi}{2} + 2(0)\pi\right) + i\sin\frac{1}{3}\left(\frac{\pi}{2} + 2(0)\pi\right)\right] = 2\left(\frac{\sqrt{3}}{2} + i\frac{1}{2}\right) = \sqrt{3} + 1i$

$n=1$: $z = 2\left[\cos\frac{1}{3}\left(\frac{\pi}{2} + 2(1)\pi\right) + i\sin\frac{1}{3}\left(\frac{\pi}{2} + 2(1)\pi\right)\right] = 2\left(-\frac{\sqrt{3}}{2} + i\frac{1}{2}\right) = -\sqrt{3} + 1i$

$n=2$: $z = 2\left[\cos\frac{1}{3}\left(\frac{\pi}{2} + 2(0)\pi\right) + i\sin\frac{1}{3}\left(\frac{\pi}{2} + 2(0)\pi\right)\right] = 2(0 - 1i) = -2i$

Solutions: $z = \sqrt{3} + i,\ -\sqrt{3} + i,\ -2i$