

Differentiation Questions

Handout 1 Differentiation by Rule

LCHL

Section 6C

p.184

$$Q2 \quad (i) \quad f(x) = 7x^2 - \frac{3}{x} = 7x^2 - 3x^{-1}$$

$$f'(x) = 14x + 3x^{-2}$$

$$(ii) \quad f(x) = 3\sqrt{x} = 3x^{\frac{1}{2}}$$

$$f'(x) = \frac{3}{2}x^{-\frac{1}{2}}$$

$$(iii) \quad f(x) = 2\sqrt{x} + \frac{2}{x^2} = 2x^{\frac{1}{2}} + 2x^{-2}$$

$$f'(x) = x^{-\frac{1}{2}} - 4x^{-3}$$

Q2

$$(iv) f(x) = x^2 - 5\sqrt{x} = x^2 - 5x^{\frac{1}{2}}$$

$$f'(x) = 2x - \frac{5}{2}x^{-\frac{1}{2}}$$

$$(v) f(x) = \frac{3}{\sqrt{x}} = 3x^{-\frac{1}{2}}$$

$$f'(x) = -\frac{3}{2}x^{-\frac{3}{2}}$$

$$(vi) f(x) = 3x^{-2} + \frac{1}{25x} = 3x^{-2} + \frac{1}{2}x^{-\frac{1}{2}}$$

$$f'(x) = -6x^{-3} - \frac{1}{4}x^{-\frac{3}{2}}$$

Q3

$$y = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 6x$$

$$\frac{dy}{dx} = x^2 + x - 6$$

Q4

$$(i) f(x) = \sqrt[3]{x} = x^{\frac{1}{3}}$$

$$f'(x) = \frac{1}{3} x^{-2/3}$$

$$(ii) f(x) = 3\sqrt{x} - \frac{1}{x^2} = 3x^{\frac{1}{2}} - x^{-2}$$

$$f'(x) = \frac{3}{2} x^{-1/2} + 2x^{-3}$$

$$(iii) f(x) = \frac{4}{x} + \frac{3}{\sqrt{x}} = 4x^{-1} + 3x^{-1/2}$$

$$f'(x) = -4x^{-2} - \frac{3}{2} x^{-3/2}$$

Q4

$$(iv) f(x) = 6 - \frac{3}{x} = 6 - 3x^{-1}$$

$$f'(x) = 6 + 3x^{-2}$$

$$(v) f(x) = 2\sqrt{x} + \sqrt[3]{x} = 2x^{\frac{1}{2}} + x^{\frac{1}{3}}$$

$$f'(x) = x^{-\frac{1}{2}} + \frac{1}{3} x^{-2/3}$$

$$(vi) f(x) = x^2 + 3 - \frac{4}{x^{-2}} = x^2 + 3 - 4x^2 = 3 - 3x^2$$

$$f'(x) = -6x$$

Q5 $y = \sqrt{x}(1 + \sqrt{x})$ Remove brackets then find $\frac{dy}{dx}$

(i) $y = \sqrt{x} + \sqrt{x}\sqrt{x} = \sqrt{x} + x = x^{\frac{1}{2}} + x$

$$\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} + 1 = \frac{1}{2\sqrt{x}} + 1$$

(ii) $\frac{dy}{dx}(x=4) = \frac{1}{2\sqrt{4}} + 1 = \frac{1}{4} + 1 = 1\frac{1}{4}$

Q6 $f(x) = x^3 + 2\sqrt{x} = x^3 + 2x^{\frac{1}{2}}$

$$f'(x) = 3x^2 + x^{-\frac{1}{2}} = 3x^2 + \frac{1}{\sqrt{x}}$$

$$f'(4) = 3(4)^2 + \frac{1}{\sqrt{4}} = 48 + \frac{1}{2} = 48\frac{1}{2}$$

Q7

$$f(x) = \frac{1}{\sqrt{x}} = x^{-1/2}$$

$$f'(x) = -\frac{1}{2} x^{-3/2} = -\frac{1}{2(\sqrt{x})^3}$$

$$f'(4) = -\frac{1}{2(\sqrt{4})^3} = -\frac{1}{2(2)^3} = -\frac{1}{16}$$

Q8

$$y = x^{5/2}$$

$$\frac{dy}{dx} = \frac{5}{2} x^{3/2}$$

$$\frac{dy}{dx}(x=2) = \frac{5}{2} (2)^{3/2} = \frac{5(2\sqrt{2})}{2} = 5\sqrt{2}$$

Q9

$$f(x) = x^2 + kx$$

$$f'(1) = 3, \quad k = ?$$

$$f'(x) = 2x + k$$

$$f'(1) = 2(1) + k = 2 + k$$

$$f'(1) = 3 \quad \Rightarrow \quad 2 + k = 3$$

$$k = 1$$

Q10

$$y = \sqrt{x} + \frac{1}{\sqrt{x}} \quad \text{show} \quad \frac{dy}{dx} = \frac{x-1}{2x\sqrt{x}}$$

$$y = x^{\frac{1}{2}} + x^{-\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} - \frac{1}{2}x^{-\frac{3}{2}} = \frac{1}{2\sqrt{x}} - \frac{1}{2(\sqrt{x})^3}$$

$$\text{note: } (\sqrt{x})^3 \Rightarrow (\sqrt{x})(\sqrt{x})(\sqrt{x}) = x\sqrt{x} = \frac{1}{2\sqrt{x}} - \frac{1}{2x\sqrt{x}}$$

$$\text{LCM} \rightarrow 2x\sqrt{x}$$

$$= \frac{x-1}{2x\sqrt{x}}$$

Q11

Find slope of tangent to $y = x^2 - 2x - 3$
at $(2, 3)$

$$\frac{dy}{dx} = 2x - 2$$

$$\frac{dy}{dx} (x=2) = 2(2) - 2 = 2$$

Q12

Find slope of $y = 2x^3 - 3x + 4$ at $(1, 3)$

$$\frac{dy}{dx} = 6x^2 - 3$$

$$\frac{dy}{dx} (x=1) = 6(1)^2 - 3 = 3$$

Q13

Slope of $y = 6 + x - x^2$ at $(2, 4) = ?$

$$\frac{dy}{dx} = 1 - 2x$$

$$\frac{dy}{dx} (x=2) = 1 - 2(2) = -3$$

Q14

 $y = 8 + 2x - x^2$ slope = 6, $x = ?$

$$\frac{dy}{dx} = 2 - 2x$$

$$\Rightarrow 2 - 2x = 6$$

$$-2x = 4$$

$$x = -2$$

Q15 $y = x^2 - x$ slope = 1 @ what point?

$$\frac{dy}{dx} = 2x - 1$$

$$\Rightarrow 2x - 1 = 1$$

$$2x = 2$$

$$x = 1$$

$$y = (1)^2 - (1) = 0$$

pt (1, 0)

Q16

$y = 2x^2 - x - 4$ Find point when slope = 3

$$\frac{dy}{dx} = 4x - 1 \quad \Rightarrow 4x - 1 = 3$$

$$4x = 4$$

$$x = 1$$

$$y = 2(1)^2 - 1 - 4 = -3$$

pt (1, -3)

Q17

$$y = x^2 + ax$$

when $x = -1$ slope = 3Find a ?

$$\frac{dy}{dx} = 2x + a$$

$$\frac{dy}{dx}(x=-1) = 2(-1) + a = -2 + a$$

$$\Rightarrow -2 + a = 3$$

$$a = 5$$

Q. 18

$$y = x^2 - 3x + 4$$

show tangent is // to
X-axis when $x = \frac{1}{2}$

X-axis has slope of 0

$$\frac{dy}{dx} = 2x - 3$$

$$\frac{dy}{dx}(x=\frac{3}{2}) = 2(\frac{3}{2}) - 3 = 0$$

QED

Q19

$$y = 2x^2 - 8x + 3$$

tangent // to $4x - y + 2 = 0$

Find point?

$$\text{slope of line} = \frac{-4}{-1} = 4$$

$$\frac{dy}{dx} = 4x - 8 \Rightarrow 4x - 8 = 4 \Rightarrow 4x = 12 \Rightarrow x = 3$$

$$y = 2(3)^2 - 8(3) + 3 = -3$$

$$\text{pt } (3, -3)$$

Q20

$$y = 2x + \frac{50}{x}$$

find x if $\frac{dy}{dx} = 0$

$$y = 2x + 50x^{-1}$$

$$\frac{dy}{dx} = 2 - 50x^{-2}$$

$$\Rightarrow 2 - \frac{50}{x^2} = 0$$

$$+\frac{50}{x^2} = +2$$

$$50 = 2x^2$$

$$25 = x^2 \Rightarrow x = \pm 5$$

Q21

$$y = a\sqrt{x} + b$$

$$\frac{dy}{dx} = 3 \quad \text{at } (4, b)$$

Find a and b ?

$$y = ax^{\frac{1}{2}} + b$$

$$\frac{dy}{dx} = \frac{1}{2}ax^{-\frac{1}{2}} \Rightarrow \frac{dy}{dx}(x=4) = \frac{1}{2}a(4)^{-\frac{1}{2}} = \frac{a}{4}$$

$$\Rightarrow \frac{a}{4} = 3 \Rightarrow a = 3(4) = 12$$

 $b = ?$

$$b = 12\sqrt{4} + b$$

$$b = 24 + b$$

$$b = -18$$

$$y = (x-1)^{3/2} - 3(x-1)^{1/2}$$

$$\text{Show } \frac{dy}{dx} = \frac{3(x-2)}{2\sqrt{x-1}}$$

$$\frac{dy}{dx} = \frac{3}{2}(x-1)^{1/2} \cdot (1) - \frac{3}{2}(x-1)^{-1/2} (1)$$

$$= \frac{3\sqrt{x-1}}{2} - \frac{3}{2\sqrt{x-1}}$$

$$= \frac{3(x-1) - 3}{2\sqrt{x-1}} = \frac{3x - 3 - 3}{2\sqrt{x-1}} = \frac{3x - 6}{2\sqrt{x-1}} = \frac{3(x-2)}{2\sqrt{x-1}}$$

QED