Differentiation Revision Questions

1. Limits and continuity

e.g. The function $f:\mathbb{R}\to\mathbb{R}$ is defined by

$$f:x\to\begin{cases}\dfrac{x^2-4}{x-2}, & \text{for } x<2\\ 3, & \text{for } x=2\\ \dfrac{3x-2}{x-1}, & \text{for } x>2\end{cases}$$

(i) What is $f(2)$?

(ii) Determine if $\lim_{x\to 2} f(x)$ exists.

(iii) Determine if *f* is continuous at $x=2$.

2. Theory of differentiation

e.g. The function *f* is defined for all $x\in\mathbb{R}$ by $f:x\to x^2-2x+4$.

(i) Find, from first principles, the derivative of $y=f(x)$.

(ii) The derivative is sometimes described as the 'slope function'. Explain what this means in the light of this derivative.

(iii) Find the instantaneous rate of change of *f* at $x=3$.

(iv) Find the average rate of change of *f* over the interval from $x=3$ to $x=4$.

3. Differentiation by rule

e.g. find $\frac{dy}{dx}$ if

(i) $y = \tan^{-1}\left(\frac{x}{\sqrt{9-x^2}}\right)$

(ii) $y = e^{1+2\sin x}(1-2\cos x)$

(iii) $y = \ln\left(x^2\sqrt{x^3+2}\right)$

4. Curve sketching

e.g. $f : x \to \frac{e^x}{e^x+1}$ is a function defined for all $x \in \mathbb{R}$.

(i) Find the horizontal asymptotes of the curve $y = f(x)$.

(ii) Show that the curve $y = f(x)$ has no turning points.

(iii) Find the co-ordinates of the point of inflection of the curve $y = f(x)$ and determine where the curve is concave upwards, and where it is concave downwards.

(iv) Sketch a rough graph of the curve $y = f(x)$.

5. **Maximum and minimum problems**
e.g. A car retailer has calculated a cost function, $C(q)$, which expresses the annual cost of purchasing and maintaining his stock of cars as a function of q, the number of cars ordered each time the stock is renewed. This is given by:

$$C(q) = \frac{4860}{q} + 15q + 750000,$$

where C is in euro.
(i) Determine the value of q which minimises the cost function.
(ii) What is the minimum value of the cost function?

6. **Rates of change**
e.g. Sand is being poured at a rate of $100 \text{ cm}^3\text{s}^{-1}$ to form a conical pile whose height is always three times the radius of its base. Find the rate at which the radius of the conical pile is changing when its height is 48 cm.