Leaving Cert

Higher Level
Project Maths

Differentiation

Find the derivative of: (i) $y = 2x^5 + x^3$ (ii) $y = \frac{1}{x^4}$ (with respect to x)

$f(x)$ $f'(x)$

x^n nx^{n-1}

(i) $\frac{dy}{dx} = 10x^4 + 3x^2$ ✓

rewrite as power

(ii) $y = \frac{1}{x^4} = x^{-4}$

$\frac{dy}{dx} = -4x^{-5}$ ✓

If $f(x) = 8 + x^2 - \frac{1}{x}$ Find $f'(x)$

$f(x) = 8 + x^2 - x^{-1}$

$f'(x) = 2x + x^{-2}$ ✓

Find the derivative of: $\sqrt{x}(x+2)$ (with respect to x)

Product rule

$y = uv \Rightarrow \frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

$f'(x) = (\sqrt{x})(1) + (x+2)(\frac{1}{2}x^{-\frac{1}{2}})$

$= \sqrt{x} + \frac{(x+2)}{2\sqrt{x}}$ ✓

Find the derivative of y = $\dfrac{1}{2+5x}$ (with respect to x)

Rewrite as power and apply chain Rule

$$y = (2+5x)^{-1}$$

$$\frac{dy}{dx} = -1(2+5x)^{-2}(5)$$

$$= \frac{-5}{(2+5x)^2}$$

Find the derivative of y = $\cos^4 x$ (with respect to x)

Chain Rule

$f(x)$	$f'(x)$
$\cos x$	$-\sin x$

$$y = \cos^4 x = (\cos x)^4$$

$$\frac{dy}{dx} = 4(\cos x)^3(-\sin x)$$

$$= -4\sin x \cos^3 x$$

Find the derivative of y = $\sin^{-1}\frac{x}{5}$ (with respect to x)

inverse trig. function

$f(x)$	$f'(x)$
$\sin^{-1}\frac{x}{a}$	$\frac{1}{\sqrt{a^2-x^2}}$

$a=5$

$$\frac{dy}{dx} = \frac{1}{\sqrt{25-x^2}}$$ ✓

Find the derivative of $y = 2x - \sin 2x$ (with respect to x)

use chain rule on $-\sin 2x$!

$f(x)$	$f'(x)$
$\sin x$	$\cos x$

$$\frac{dy}{dx} = 2 - (\cos 2x)(2)$$

$$= 2 - 2\cos 2x$$ ✓

Find the derivative of $y=\ln\left(x^2+1\right)$ (with respect to x)

$f(x)$	$f'(x)$
$\ln x$	$\frac{1}{x}$

Chain Rule

$$\frac{dy}{dx} = \frac{1}{(x^2+1)}\cdot(2x)$$

$$= \frac{2x}{x^2+1} \checkmark$$

Find the derivative of $y=\left(\dfrac{3+x}{\sqrt{9-x^2}}\right)$ (with respect to x)

Quotient rule

$$y=\frac{u}{v} \Rightarrow \frac{dy}{dx}=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$$

$u = 3+x$

$\frac{du}{dx} = 1$

$v = \sqrt{9-x^2} = (9-x^2)^{\frac{1}{2}}$

$\frac{dv}{dx} = \frac{1}{2}(9-x^2)^{-\frac{1}{2}}(-2x)$

$= \frac{-x}{\sqrt{9-x^2}}$

$$\frac{dy}{dx} = \frac{(\sqrt{9-x^2})(1) - (3+x)\left(-\frac{x}{\sqrt{9-x^2}}\right)}{(\sqrt{9-x^2})^2} \checkmark$$

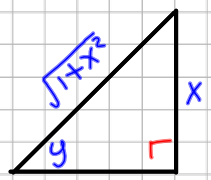
$$= \frac{\sqrt{9-x^2} + \frac{x(3+x)}{\sqrt{9-x^2}}}{(\sqrt{9-x^2})^2}\cdot\frac{(\sqrt{9-x^2})}{(\sqrt{9-x^2})}$$

$$= \frac{9-x^2 + x(3+x)}{(\sqrt{9-x^2})^3}$$

$$= \frac{9-\cancel{x^2}+3x+\cancel{x^2}}{(\sqrt{9-x^2})^3}$$

$$= \frac{3x+9}{(\sqrt{9-x^2})^3}$$

Find the derivative of $y = \sin^{-1}\left(\dfrac{x}{\sqrt{1+x^2}}\right)$ (with respect to x)



$\tan y = \frac{x}{1}$

$\left(\sqrt{1+x^2}\right)^2 - x^2 = 1$

$y = \tan^{-1}\left(\frac{x}{1}\right)$

$f(x)$	$f'(x)$
$\tan^{-1}\frac{x}{a}$	$\frac{a}{a^2+x^2}$

If $y = \sin x \cos x$ find the slope of the curve when $x = \dfrac{\pi}{2}$

Product rule

$$y = uv \quad \Rightarrow \quad \frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{dy}{dx} = (\sin x)(-\sin x) + (\cos x)(\cos x)$$
$$= \cos^2 x - \sin^2 x$$

Sub in Value

$$x = \frac{\pi}{2} \Rightarrow \frac{dy}{dx} = \cos^2\frac{\pi}{2} - \sin^2\frac{\pi}{2}$$
$$= -1$$

If $y = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$ Show $\dfrac{dy}{dx} = \dfrac{4}{(e^x + e^{-x})^2}$

Quotient rule

$$y = \frac{u}{v} \Rightarrow \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$u = e^x - e^{-x}$

$\dfrac{du}{dx} = e^x + e^{-x}$

$v = e^x + e^{-x}$

$\dfrac{dv}{dx} = e^x - e^{-x}$

note: $e^x e^{-x} = 1$

$$\frac{dy}{dx} = \frac{(e^x+e^{-x})(e^x+e^{-x}) - (e^x-e^{-x})(e^x-e^{-x})}{(e^x+e^{-x})^2}$$

$$= \frac{(e^x+e^{-x})^2 - (e^x-e^{-x})^2}{(e^x+e^{-x})^2}$$

(DOTS)

$$= \frac{[\cancel{e^{2x}} + 2e^x e^{-x} + \cancel{e^{-2x}}] - [\cancel{e^{2x}} - 2e^x e^{-x} + \cancel{e^{-2x}}]}{(e^x+e^{-x})^2}$$

$$= \frac{2-(-2)}{(e^x+e^{-x})^2} = \frac{4}{(e^x-e^{-x})^2}$$

QED

If $f(x) = 3\cos(2x+5)$, show that $f''(x) + 4f(x) = 0$

Chain Rule

$f(x)$	$f'(x)$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$

$$f'(x) = 3[-\sin(2x+5)](2)$$
$$= -6\sin(2x+5)$$

$$f''(x) = -6[\cos(2x+5)](2)$$
$$= -12\cos(2x+5)$$

$\Rightarrow f''(x) + 4f(x)$

$= -12\cos(2x+5) + 4[3\cos(2x+5)]$

$= 0$

QED

Find the slope of the tangent to the circle $x^2 + y^2 = 25$ at the point $(3, -4)$.

Differentiate w.r.t x

$$2x + 2y\frac{dy}{dx} = 0$$

$$2y\frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{2y} = -\frac{x}{y}$$

at $(3, -4)$

$$\frac{dy}{dx} = \frac{-3}{-4} = \frac{3}{4}$$

note:

$f(x)$	$f'(x)$
y	$\frac{dy}{dx}$
y^2	$2y\frac{dy}{dx}$