Leaving Cert

Higher Level
Project Maths

Differentiation

The Syllabus

	Ordinary level	Higher level
5.2 Calculus	– find first and second derivatives of linear, quadratic and cubic functions by rule – associate derivatives with slopes and tangent lines – apply differentiation to • rates of change • maxima and minima • curve sketching	– differentiate linear and quadratic functions from first principles – differentiate the following functions • polynomial • exponential • trigonometric • rational powers • inverse functions • logarithms – find the derivatives of sums, differences, products, quotients and compositions of functions of the above form – apply the differentiation of above functions to solve problems – use differentiation to find the slope of a tangent to a circle

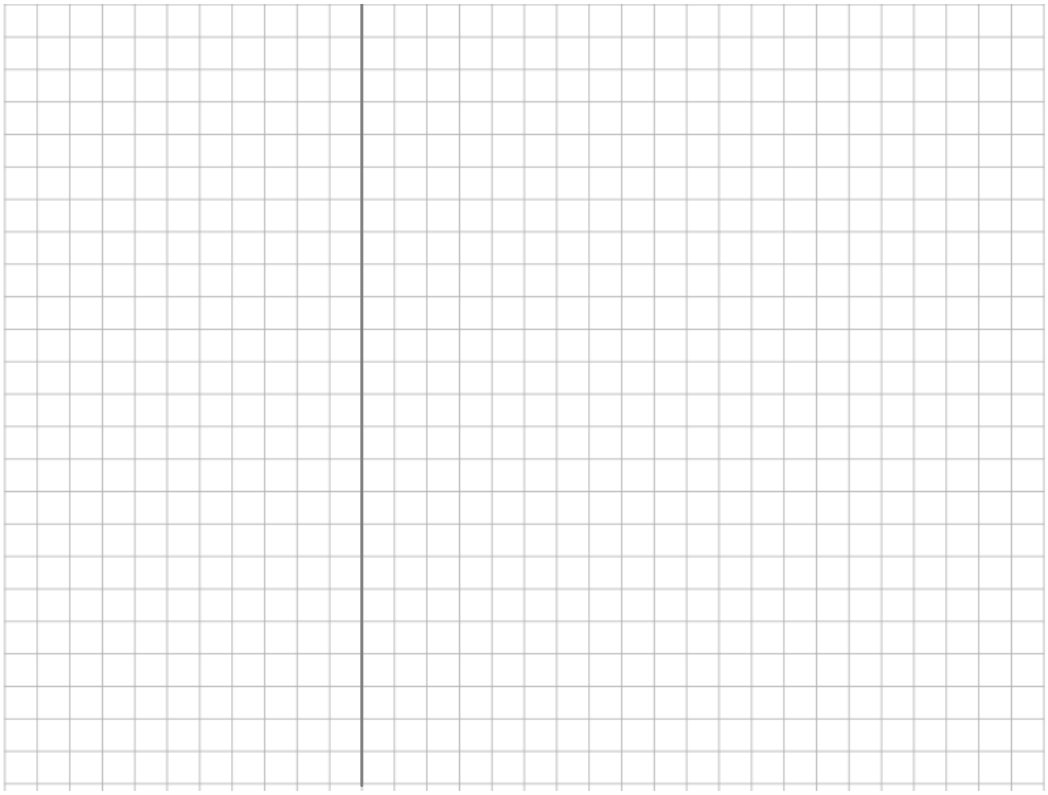
Differerentiation Revision

Find the derivative of: (i) $y = 2x^5 + x^3$ (ii) $y = \frac{1}{x^4}$ (with respect to x)

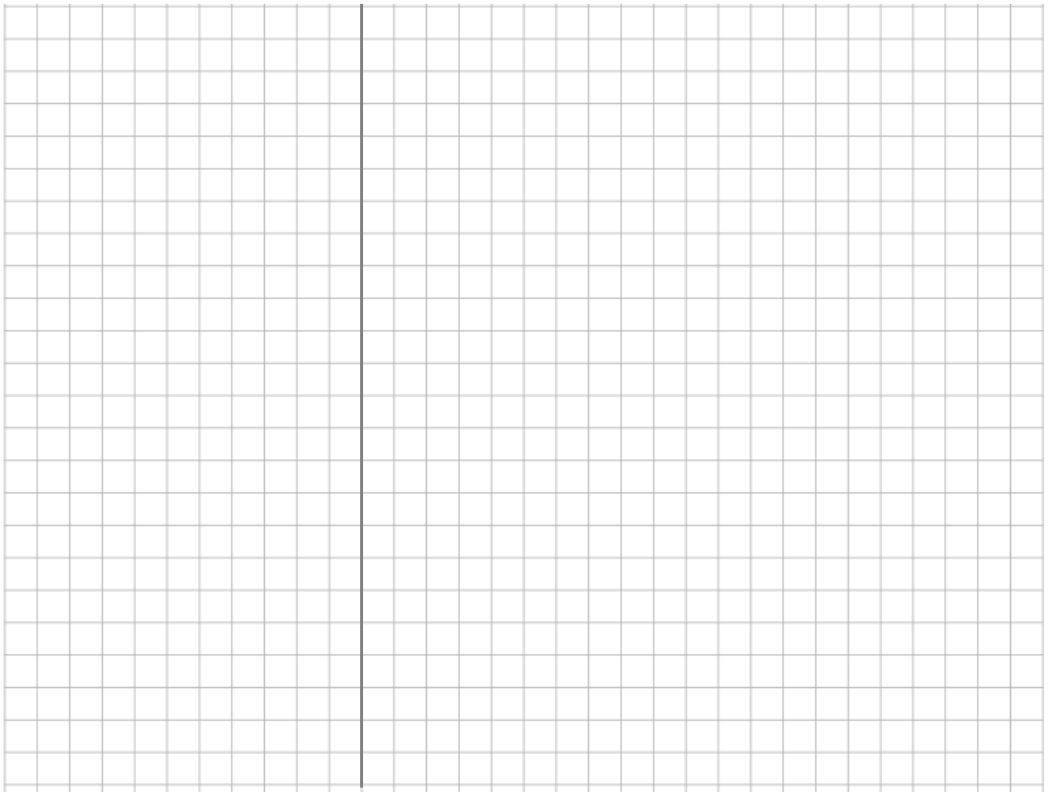
If $f(x) = 8 + x^2 - \frac{1}{x}$ Find $f'(x)$

Differerentiation Revision

Find the derivative of: $\sqrt{x}\left(x+2\right)$ (with respect to x)



Find the derivative of y = $\frac{1}{2+5x}$ (with respect to x)



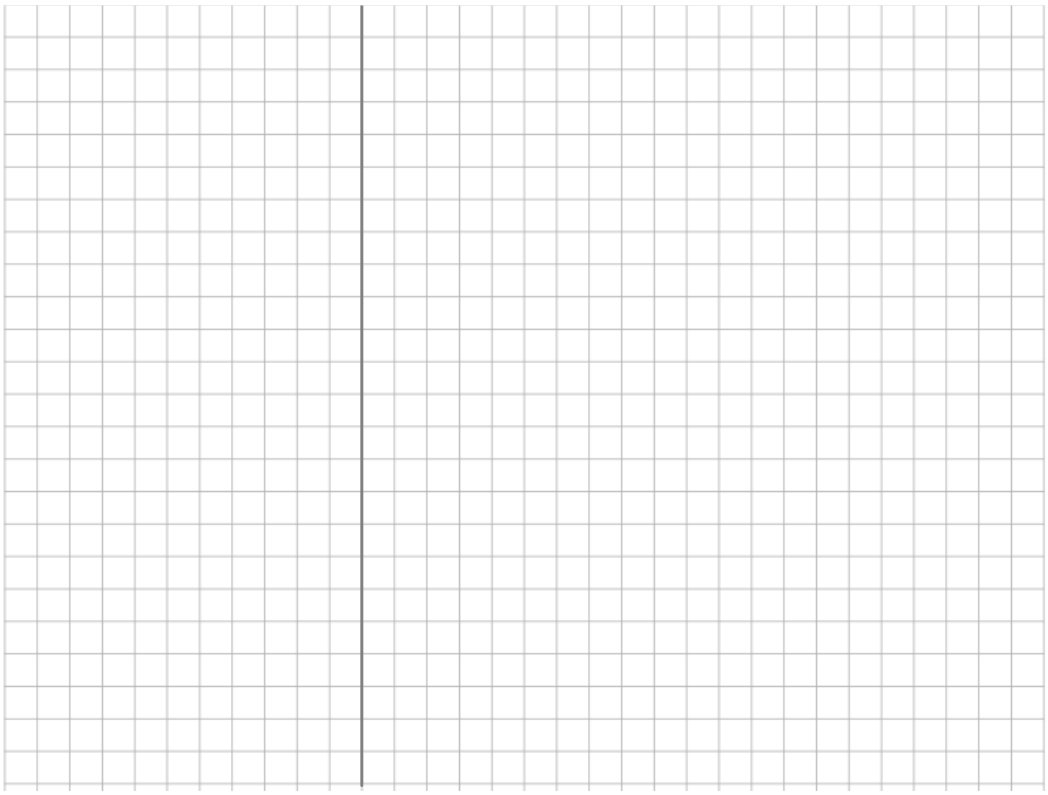
Differerentiation Revision

Find the derivative of y = $\cos^4 x$ (with respect to x)

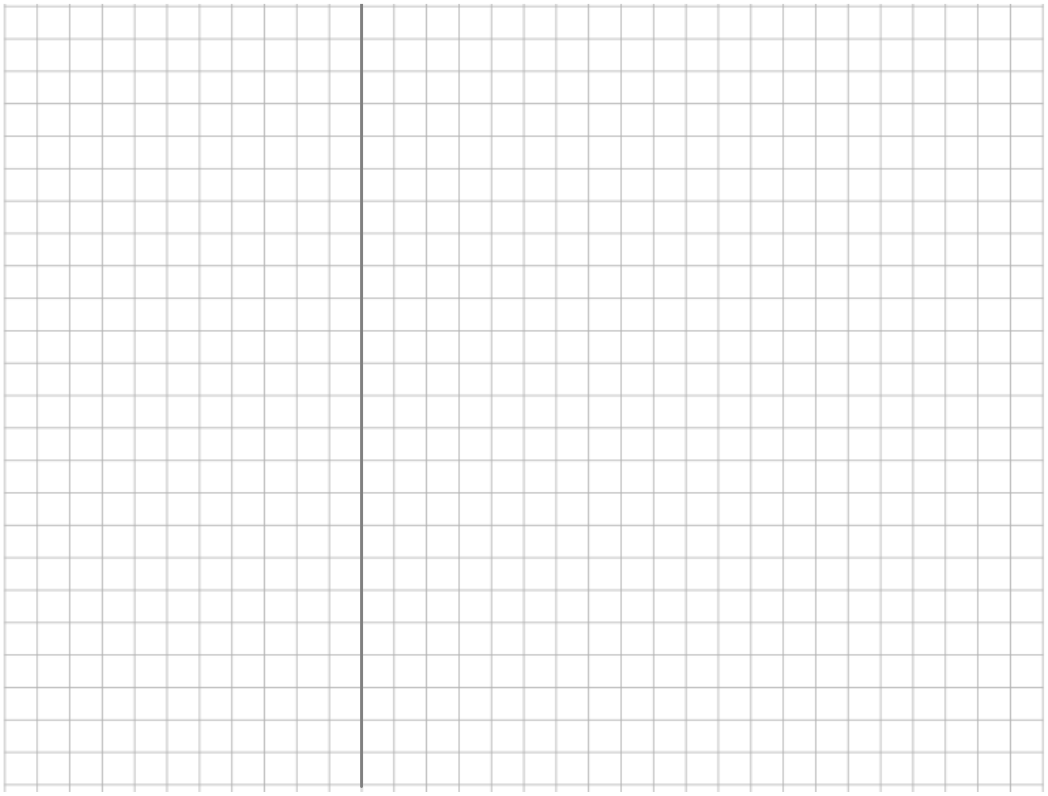
Find the derivative of y = $\sin^{-1}\frac{x}{5}$ (with respect to x)

Differerentiation Revision

Find the derivative of $y = 2x - \sin 2x$ (with respect to x)

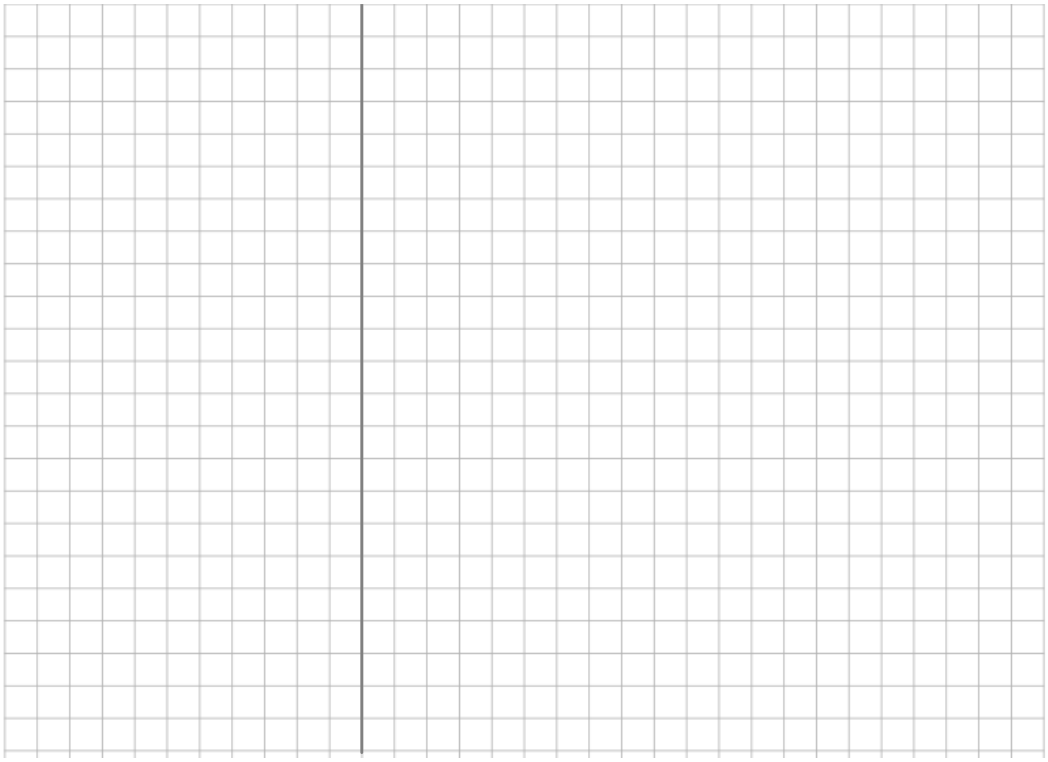


Find the derivative of $y = \ln\left(x^2 + 1\right)$ (with respect to x)

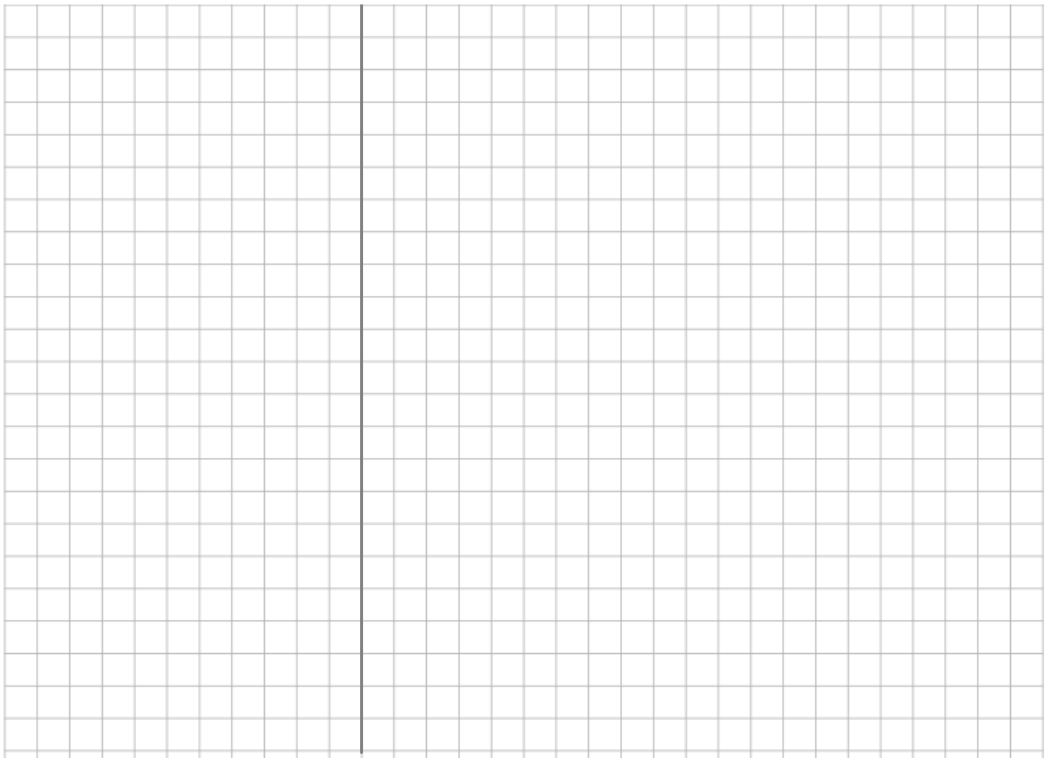


Differerentiation Revision

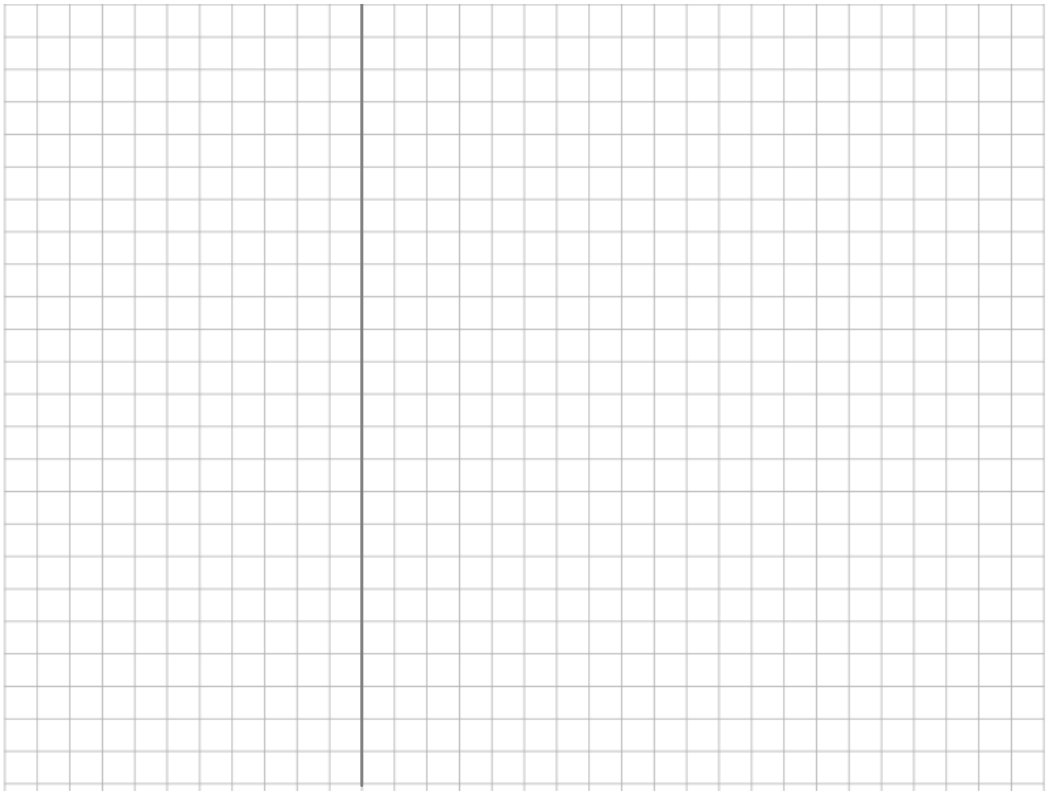
Find the derivative of $y = \left(\frac{3+x}{\sqrt{9-x^2}} \right)$ (with respect to x)



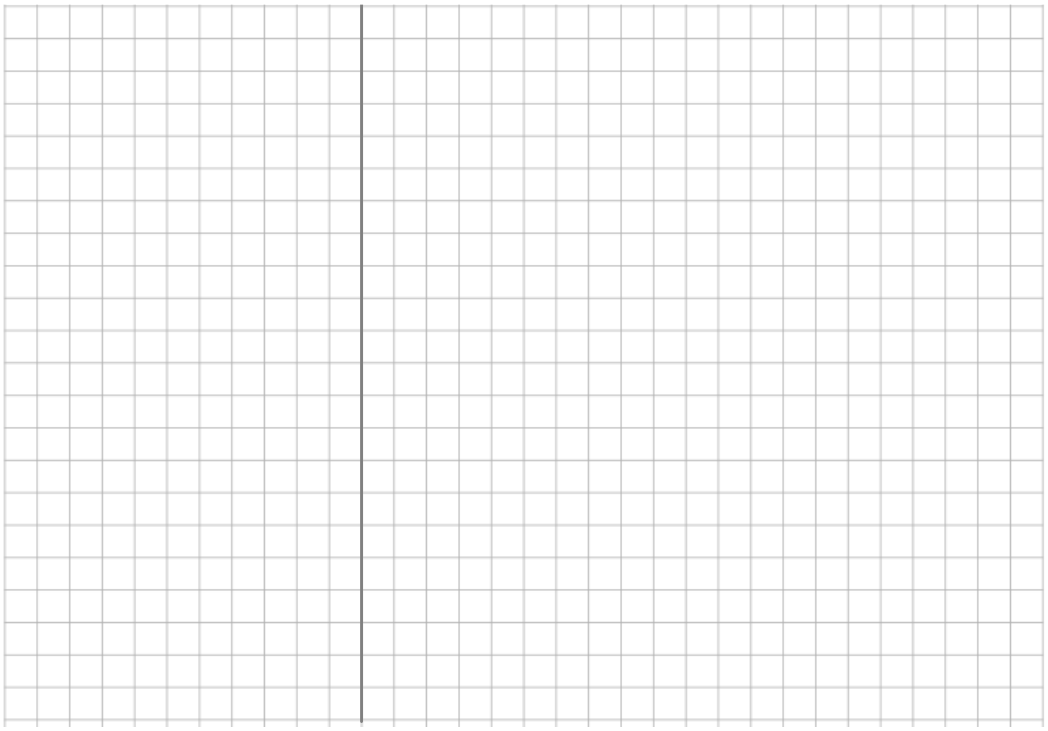
Find the derivative of $y = \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}} \right)$ (with respect to x)



If $y = \sin x \cos x$ find the slope of the curve when $x = \frac{\pi}{2}$



If $y = \frac{e^x - e^{-x}}{e^x + e^{-x}}$ Show $\frac{dy}{dx} = \frac{4}{(e^x + e^{-x})^2}$



If $f(x) = 3\cos(2x+5)$, show that $f''(x) + 4f(x) = 0$

Find the slope of the tangent to the circle $x^2 + y^2 = 25$ at the point $(3,-4)$.

$y = \tan^{-1}\left(\frac{x}{\sqrt{9-x^2}}\right)$ $\frac{dy}{dx} = ?$

(ii) $y = e^{1+2\sin x}(1-2\cos x)$ $\frac{dy}{dx} = ?$

(iii) $y = \ln\left(x^2\sqrt{x^3+2}\right)$ $\quad \frac{dy}{dx} = ?$

$f(x) = x^3 + 2x^2 - 4x + 3$

Find the local max, local min, and point of inflection?

Differentiation

$f(x)$	$f'(x)$
x^n	nx^{n-1}
$\ln x$	$\frac{1}{x}$
e^x	e^x
e^{ax}	ae^{ax}
a^x	$a^x \ln a$
$\cos x$	$-\sin x$
$\sin x$	$\cos x$
$\tan x$	$\sec^2 x$
$\cos^{-1}\frac{x}{a}$	$-\frac{1}{\sqrt{a^2-x^2}}$
$\sin^{-1}\frac{x}{a}$	$\frac{1}{\sqrt{a^2-x^2}}$
$\tan^{-1}\frac{x}{a}$	$\frac{a}{a^2+x^2}$

Product rule

$$y = uv \quad \Rightarrow \quad \frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

Chain rule

$$y = u(v(x)) \quad \Rightarrow \quad \frac{dy}{dx} = \frac{du}{dv}\frac{dv}{dx}$$

Quotient rule

$$y = \frac{u}{v} \quad \Rightarrow \quad \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

formula and tables p. 25

You need to know that for a function $f(x)$:

① The derivative of a function is called the "slope function" it gives us the slope of the curve for every x value. $f'(x)$ = slope

② At max and/or min values of a curve the slope = 0. At max/min $f'(x) = 0$
(also true for saddle pt.)

③ At a max point $f''(x) < 0$
and at a min point $f''(x) > 0$

④ At the point of inflection the second derivative = 0
At inflection point $f'(x) = 0$