The Factor Theorem

" If $f(k) = 0$ then $(x-k)$ is a factor"

"k is a solution"

Because of the factor theorem if we know a solution we know a factor and visa versa.

We can also use the factor theorem to Solve Cubic Equations

13. Given $f(x) = 2x^3 + 13x^2 + 13x - 10$. Ch.2.9

Show that $f(-2) = 0$ and hence find the three factors of $f(x)$.

$f(-2) = 2(-2)^3 + 13(-2)^2 + 13(-2) - 10 = -16 + 52 - 26 - 10 = 0$ ✓

$\Rightarrow (x+2)$ is a factor

DIVIDE

$$\begin{array}{r l}
 & 2x^3 + 9x - 5 \\
x+2 \,\big) & 2x^3 + 13x^2 + 13x - 10 \\
 & \mp 2x^3 \mp 4x^2 \\
\hline
 & 9x^2 + 13x \\
 & \mp 9x^2 \mp 18x \\
\hline
 & -5x - 10 \\
 & \pm 5x \pm 10 \\
\hline
\end{array}$$

FACTORISE $2x^3 + 9x - 5$

$(2x - 1)(x + 5)$

FACTORS ARE:

$(x+2)(2x-1)(x+5)$

17. Solve each of the following equations

(i) $x^3 - 4x^2 - x + 4 = 0$ (ii) $x^3 + 2x^2 - 11x - 12 = 0$

(iii) $3x^3 - 4x^2 - 3x + 4 = 0$ (iv) $x^3 - 7x - 6. = 0$

$$f(1) = (1)^3 - 4(1)^2 - (1) + 4 = 1 - 4 - 1 + 4 = 0$$

$\Rightarrow (x-1)$ is FACTOR

DIVIDE

$$\begin{array}{r|l} & x^2 - 3x - 4 \\ x-1 & x^3 - 4x^2 - x + 4 \\ & \mp x^3 \pm x^2 \\ \hline & -3x^2 - x \\ & \pm 3x^2 \mp 3x \\ \hline & -4x + 4 \\ & \pm 4x \mp 4 \\ \hline \end{array}$$

FACTORISE

$x^2 - 3x - 4$

$(x-4)(x+3)$

FACTORS:

$(x-1)(x-4)(x+3)$

17. Solve each of the following equations

(i) $x^3 - 4x^2 - x + 4 = 0$ (ii) $x^3 + 2x^2 - 11x - 12 = 0$

(iii) $3x^3 - 4x^2 - 3x + 4 = 0$ (iv) $x^3 - 7x - 6. = 0$

$$f(-1) = (-1)^3 + 2(-1)^2 - 11(-1) - 12 = -1 + 2 + 11 - 12 = 0$$

$\Rightarrow (x+1)$ IS FACTOR

DIVIDE

$$\begin{array}{r|l} & x^2 + x - 12 \\ x+1 & x^3 + 2x^2 - 11x - 12 \\ & \mp x^3 \mp x^2 \\ \hline & x^2 - 11x \\ & \mp x^2 \mp 1x \\ \hline & -12x - 12 \\ & \pm 12x \pm 12 \end{array}$$

FACTORISE

$x^2 + x - 12$

$(x-3)(x+4)$

FACTORS ARE:

$(x+1)(x-3)(x+4)$

17. Solve each of the following equations

(i) $x^3 - 4x^2 - x + 4 = 0$ (ii) $x^3 + 2x^2 - 11x - 12 = 0$

(iii) $3x^3 - 4x^2 - 3x + 4 = 0$ (iv) $x^3 - 7x - 6. = 0$

$f(1) = 3(1)^3 - 4(1)^2 - 3(1) + 4 = 3 - 4 - 3 + 4 = 0$

$\Rightarrow (x-1)$ IS FACTOR

DIVIDE

$3x^2 - x - 4$

$x-1 \overline{)\, 3x^3 - 4x^2 - 3x + 4}$

$\mp 3x^3 \pm 3x^2$

$-x^2 - 3x$

$\pm x^2 \mp 1x$

$-4x + 4$

$\pm 4x \mp 4$

FACTORISE

$3x^2 - x - 4$

$(3x - 4)(x + 1)$

FACTORS ARE:

$(x-1)(3x-4)(x+1)$

17. Solve each of the following equations

(i) $x^3 - 4x^2 - x + 4 = 0$ (ii) $x^3 + 2x^2 - 11x - 12 = 0$

(iii) $3x^3 - 4x^2 - 3x + 4 = 0$ (iv) $x^3 - 7x - 6. = 0$

$f(-1) = (-1)^3 - 7(-1) - 6 = -1 + 7 - 6 = 0$

$\Rightarrow (x+1)$ IS FACTOR

DIVIDE

$x^2 - 1x - 6$

$x+1 \overline{)\, x^3 + 0x^2 - 7x - 6}$

$\mp x^3 \mp 1x^2$

$-1x^2 - 7x$

$\pm 1x^2 \pm 1x$

$-6x - 6$

$\pm 6x \pm 6$

FACTORISE

$x^2 - x - 6$

$(x - 3)(x + 2)$

FACTORS ARE:

$(x+1)(x-3)(x+2)$

18. If $(x+1)$ and $(x+3)$ are both factors of $2x^3+ax^2+bx-3$, find the values of a and b.
Find the third factor and hence solve the equation $2x^3+ax^2+bx-3=0$.

$$f(-1)=0=2(-1)^3+a(-1)^2+b(-1)-3$$

$$\Rightarrow -2+a-b-3=0$$

$$a-b=5 \quad ①$$

$$f(-3)=0=2(-3)^3+a(-3)^2+b(-3)-3$$

$$\Rightarrow -54+9a-3b-3=0$$

$$9a-3b=-57$$

$$\Rightarrow 3a-b=-29 \quad ②$$

$$②-① \Rightarrow 2a=-34$$

$$a=-17$$

$$-17-b=5$$

$$b=-22$$