Financial Maths

For all of the Formulae:
F = Final Amount
P = Principal
i = Interest Rate (as a decimal)
t = time

A = Annual Repayment Amount

Compound Interest
$F = P(1+i)^t$

What sum of money invested now at 3% per annum will grow to €4,548 in 6 years.

$4{,}548 = P(1+0.03)^6$
$P = 3{,}808.88$

Interest Rates other than Annual

APR represents interest rate over a year. To find the corresponding monthly interest rate use formula.

An ad quoted an APR of 9.8%. If this loan is compounded monthly what is the rate of interest per month.

Over 1 year €1 would amount to €1.098

$F = P(1+i)^t$

$1.098 = 1(1+i)^{12}$

$(1.098)^{\frac{1}{12}} = 1 + i$

$i = 0.0078$ or 0.78%

Present Value
$P = \frac{F}{(1+i)^t}$

The inflation adjusted annual growth rate is 3% Calculate the present value of end of year cash inflows of €20,000 over the next 3 years.

End of Year 1 $\frac{20{,}000}{(1+.03)^1} = 19{,}417.48$
End of Year 2 $\frac{20{,}000}{(1+.03)^2} = 18{,}851.92$
End of Year 3 $\frac{20{,}000}{(1+.03)^3} = 18{,}302.83$

Net Present Value is the present value of all cash inflows – present value of cash outflows.

If $NPV > 0$ Invest in project
If $NPV \leq 0$ Do not invest in project

If the Initial Investment in the project is €55,000 is it worth investing?

$19{,}417.48 + 18{,}851.92 + 18{,}302.83 - 55{,}000$
$= 1{,}572.23$

Yes, it is worth investing as $NPV > 0$

Sometimes the interest rate i is called the **discount rate**

Depreciation (Reducing Balance Method)
$F = P(1-i)^t$

What is the annual rate of depreciation of a machine that cost €30,000, 5 years ago and is worth €10,000 now?

$10{,}000 = 30{,}000(1-i)^5$

$\left(\frac{10{,}000}{30{,}000}\right)^{\frac{1}{5}} = 1 - i$

$i = 0.1973$ or 19.73%

Sum of Series
$S_n = \frac{a(1-r^n)}{1-r}$

a = 1st amount
r = common ratio
n = number of payments

The New York Lotto pays 26 Annual Payments, the 1st one immediately and an increase of 4% on each payment each year with the last in 25 years. Or a cash value option of $21,500,000.

If A is the 1st Payment find in terms of A the amount of the 2nd, 3rd and 26th payments.

1st Payment = A
2nd Payment = $A(1.04)$
3rd Payment = $A(1.04)^2$
26th Payment = $A(1.04)^{25}$

The 26 payments form a Geometric Series. Find the advertised Jackpot in terms of A.

$a = A$
$r = (1.04)$

$S_{26} = \frac{A(1-1.04^{26})}{1-1.04} = 44.312A$

Find the value of A that corresponds to the Cash Value Payment of $21,500,000

$44.312A = 21{,}500{,}000$

$A = \$485{,}198.89$

Income Tax

Standard Tax $=$ Standard Rate cutoff $\times$ Standard Rate
Higher Tax $=$ Income above cutoff $\times$ Higher Rate
Gross Tax $=$ Standard Tax $+$ Higher Tax

Tax Payable $=$ Gross Tax $-$ Tax Credit

Net Income $=$ Gross Income $-$ Tax Payable

Other Deductions may have to be subtracted from net income.

- PRSI
- Universal Social Charge
- Health Insurance etc

Amortised Loan – Mortgages

$$A = P\frac{i(1+i)^t}{(1+i)^t - 1}$$

This formula is usually used for calculating loan repayments.
To use it for calculating the payment that must be made into an annuity or pension fund, the total value of the fund must be discounted back to present value.

Calculate the yearly repayments of a mortgage of €155,000 over 32 years at an AER of 3.5%

$$A = 155{,}000\frac{0.035(1+0.035)^{32}}{(1+0.035)^{32} - 1}$$

$$A = €8{,}128.34$$

Future Value of instalment savings over t instalments (eg. the future value of a savings fund with regular fixed payments)

$$F = P(1+i)^1 + P(1+i)^2 + P(1+i)^3 \ldots P(1+i)^t$$

Present Value of future payments is given by (eg. the present value of a pension fund that pays regular fixed payments)

$$PV = \frac{F}{(1+i)^1} + \frac{F}{(1+i)^2} + \frac{F}{(1+i)^3} \ldots + \frac{F}{(1+i)^t}$$

Both of the above situations can use $S_n = \frac{a(1-r^n)}{1-r}$ to calculate sums generated over a period of time.

A bond is a cash payment made to the government for an agreed number of years. In return the investor is paid a fixed sum at the end of each year. In addition the government repays the original value of the bond with the final payment.
An Annuity is a regular stream of fixed payments over period of time, taking into account the time value of money.
A **pension** is a fixed sum paid to a person over a period of time. We can calculate the amount needed to be invested now to guarantee a fixed income over a number of years by discounting each payment to its present value and summing them.
A Perpetuity is an annuity in which regular payments begin on a particular date and continue indefinitely.
Amortisation is the process of accounting for a sum of money by making it equivalent to a series of payments over time.