

Compound Interest

$$A = P(1+r)^t$$

P = Principal = Present value

A = Amount = Future Value

I = Interest = A - P

r = Rate of interest

(i = AER = Annual Equivalent Rate)

t = Time (no. of time intervals on which interest is added)

Example 1

Find the future value of €5000 invested at 4% (AER) per annum, compounded annually, for 6 years. Find also the interest earned over the period.

Future value = Amount

$$\begin{aligned}P &= \text{€}5000 \\r &= 4\% \\t &= 6 \text{ years}\end{aligned}$$

$$A = P(1+r)^t$$

$$A = 5000 (1.04)^6 = \text{€}6326.60$$

$$I = A - P$$

$$I = 6326.60 - 5000 = \text{€}1326.60$$

Example 2

An investment bond offers a return of 15% if invested for 4 years. Calculate the AER (annual equivalent rate) for this bond, correct to two places of decimals.

$$A = P(1+R)^t$$

$$\sqrt[t]{\frac{A}{P}} = 1+R$$

$$R = \sqrt[t]{\frac{A}{P}} - 1$$

$$R = ?$$

$$t = 4 \text{ years}$$

$$\text{Return} = 15\%$$

$$P = \text{imagine we invest } \text{€}100$$

$$A = \text{end we have} = \text{€}115$$

$$R = \sqrt[4]{\frac{115}{100}} - 1 = 0.035558$$

$$= 3.55\% \checkmark$$

Example 3

€5000 is invested at 4% AER. If the interest is added monthly, find the future value of this investment after (i) $3\frac{1}{2}$ years (ii) 5 years 2 months.

$$A = ?$$

$$P = \text{€}5000$$

$$i \quad R = 4\% = \text{AER}!$$

note this does not mean 4% to be added on monthly.

let i = AER and R = monthly interest rate

$$R = ?$$

$$(1+R)^{12} = (1+i)$$

$$R = \sqrt[12]{1+i} - 1$$

$$R = \sqrt[12]{1.04} - 1 = 0.003274$$

$$(i) \quad 3\frac{1}{2} \text{ years} = 12(3.5) = 42 \text{ months}$$

$$A = 5000(1.003274)^{42} = \text{€}5735.77$$

$$(ii) \quad 5 \text{ years } 2 \text{ months} = 5(12) + 2 = 62 \text{ months}$$

$$A = 5000(1.003274)^{62} = \text{€}6123.26$$

Exercise 5.1

1. Find the future value, correct to 2 places of decimals, of €3000 invested for 10 years at an annual equivalent rate (AER) of 3%.

Future value = Amount

$$A = P(1 + R)^t$$

$$P = €3000$$

$$t = 10 \text{ years}$$

$$R = 3\%$$

$$A = 3000 (1.03)^{10} = €4031.75$$

2. Given an AER of 2.5%, find the future value, correct to 2 places of decimals, of €5000 invested for 8 years. What interest would be paid on this investment?

$$A = ?$$

$$R = 2.5\%$$

$$P = €5000$$

$$t = 8 \text{ years}$$

$$A = P(1 + R)^t$$

$$A = 5000 (1.025)^8 = €6091.01$$

6. Sandra wins €15 000 in a draw and invests it in a credit union where the AER is 3.5%.

Copy and complete this chart, showing how the value of her money changes over the five years of the investment.

Year	Principal	Interest €
One	€15 000	525
Two	€15 525	1 068.38
Three	€16 068.38	1 630.77
Four	€16 630.77	2 212.85
Five	€17 212.85	2 815.30

$$A = P(1+R)^t$$

$$I = A - P$$

yr.1

$$A_1 = 15000(1.035)^1 = 15,525$$

$$I_1 = 15,525 - 15000 = 525$$

yr.2

$$A_2 = 15,525(1.035)^1 = €16,068.38$$

$$I_2 = 16,068.38 - 15000 = 1068.38$$

yr.3

$$A_3 = 16,068.38(1.035)^1 = €16,630.77$$

$$I_3 = 16,630.77 - 15000 = 1630.77$$

yr.4

$$A_4 = 16,630.77(1.035)^1 = €17,212.85$$

$$I_4 = 17,212.85 - 15000 = 2212.85$$

yr.5

$$A_5 = 17,212.85(1.035)^1 = €17,815.30$$

$$I_5 = 17,815.30 - 15000 = 2815.30$$