

15. The formula  $(1 + r)^{12} = (1 + i)$ , where  $r$  is the interest rate per month and  $i$  the interest rate per annum, is used to calculate the effective monthly interest rate.
- ✓(i) If 6% interest is offered per year, calculate the effective monthly rate correct to four places of decimals.
- (ii) If  $r$  was simply calculated by dividing the yearly interest by 12, calculate using both methods, the difference in future values of €10 000 in 3 years at 6% per annum, if the interest is compounded monthly.
- (iii) What is the minimum number of places of decimals that need to be taken in calculating  $r$  before a difference is noted in future values?

$(1+r)^{12} = (1+i)$  (i)

$$\Rightarrow 1+r = \sqrt[12]{1+i} \Rightarrow R = \sqrt[12]{1+i} - 1$$

$$R = \sqrt[12]{1.06} - 1 = 0.004867 = \text{MER}$$

$$\approx 0.005 \text{ (3 dp)}$$

3 years = 36 months

(ii)

method 1  $r = \frac{1}{12} = \frac{0.06}{12} = 0.005$

method 2  $A = 10,000 (1.005)^{36} = €11,966.81$

$A = 10,000 (1.004867)^{36} = €11,909.93$

difference = €56.88

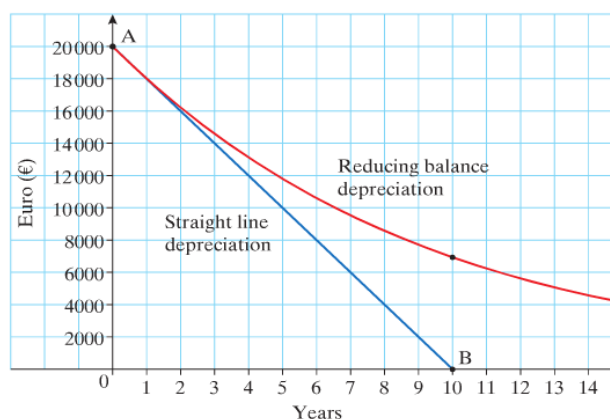
(iii) 4 decimals

## Section 5.2 Depreciation

In the previous section, money lodged into a savings account appreciated in value. The future value was greater than the present value.

Depreciation occurs when the future value of an asset is less than the present value. Cars, computers and household appliances generally depreciate in value over time. Houses in Ireland appreciated in value up to 2007 but have since greatly depreciated in value relative to this "peak" value. Two types of depreciation can be considered.

- Straight line depreciation** occurs when the value of an object reduces by a constant amount each year.  
For example, take a car costing €20 000 that loses 10% of its original value each year. This car loses €2000 in value each year and so the car has no value after 10 years.
- Reducing balance depreciation** occurs when the value of an object reduces by a fixed percentage of its value each year.  
Consider a car costing €20 000 that loses 10% of its value each year on a reducing balance. The value of the car after 10 years = €20 000  $(1 - 0.1)^{10}$  = €6973.57.



Depreciation:  $F = P(1 - i)^t$

$F$  = future value

$i$  = the percentage depreciation of €P per year

$t$  = number of years

$P$  = initial value

### Example 1

A company buys a new machine priced at €35 000.

The machine depreciates by 20% on a reducing balance basis each year.

(i) What will the value of the machine be in 4 years time?

(ii) By how much has the machine depreciated in value during this time?

$$F = P(1 - i)^t$$

$$(i) \quad F = 35\,000 (0.8)^4 = €14,336$$

$$(ii) \quad D = 35\,000 - 14\,336 = €20,664$$

### Example 2

A garage has a petrol stock of 100 000 litres.

If the manager estimates (a) that he will sell 4000 litres a day

(b) that he will sell 5% of his stock per day,

calculate the difference in his estimates after 20 days.

a)

$$100,000 - 4000(20) = 20,000 \text{ litres}$$

$$F = P(1 - i)^t$$

$$b) \quad F = 100,000 (0.95)^{20} \\ = 35,848.59 \text{ litres}$$

$$\text{Difference} = 15,849 \text{ litres}$$

4. A company buys a machine costing €140 000.  
 In order to facilitate its replacement, the company invests €25 000 in a bank offering a return of 3.5% per annum compound interest.  
 If the machine depreciates at a rate of 20% per annum, find
- (i) the value of the machine in 4 years time  
 (ii) the value of their savings investment in 4 years time.
  - If inflation over the 4 years averages 2% per annum, find
    - the cost of buying a new machine in 4 years time
    - how much money the company will need to add to their savings in order to replace the machine, taking the second-hand value of the machine in 4 years time into account.

(Note: Inflation is a rise in the *general level of prices* of goods and services in an economy.)

$$F = P(1-i)^t$$

$$A = P(1+i)^t$$

$$P = \frac{A}{(1+i)^t}$$

a) i)  $F = 140,000 (0.8)^4 = € 57,344$

ii)  $A = 25,000 (1.035)^4 = € 28,688.08$

b) i)  $A = 140,000 (1.02)^4 = € 151,540.50$

ii) Difference between depreciated value and cost of new machine in 4 years.

$$= 151,540.50 - 57,344 = € 94,196.50$$

Extra money needed = Value difference Less savings

$$= 94,196.50 - 28,688.08 = € 65,508.42$$

Extra investment to amount to this

$$P = 65,508.42 / (1.04)^4 = € 55,996.90$$