4. A company buys a machine costing €140 000.
In order to facilitate its replacement, the company invests €25 000 in a bank offering a return of 3.5% per annum compound interest.
If the machine depreciates at a rate of 20% per annum, find

(a) (i) the value of the machine in 4 years time
(ii) the value of their savings investment in 4 years time.

(b) If inflation over the 4 years averages 2% per annum, find
(i) the cost of buying a new machine in 4 years time
(ii) how much money the company will need to add to their savings in order to replace the machine, taking the second-hand value of the machine in 4 years time into account.

(**Note:** Inflation is a rise in the *general level of prices* of goods and services in an economy.)

$F=P(1-i)^t$

$A=P(1+i)^t$

$P=\dfrac{A}{(1+i)^t}$

a) i) $F = 140{,}000\,(0.8)^4 = €\,57{,}344$

ii) $A = 25{,}000\,(1.035)^4 = €\,28{,}688.08$

b) i) $A = 140{,}000(1.02)^4 = €\,151{,}540.50$

ii) Difference between depreciated value and cost of new machine in 4 years.

$= 151{,}540.50 - 57344 = €\,94{,}196.50$

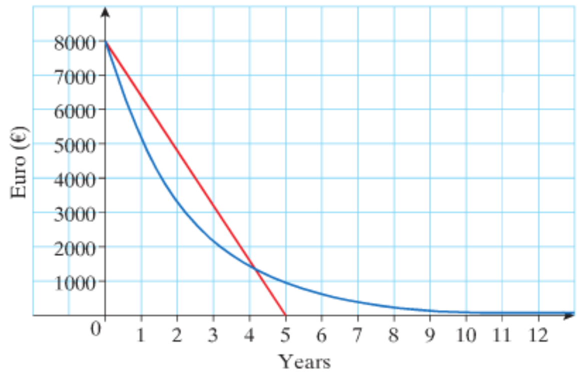
Extra money needed = Value difference Less savings

$= 94196.50 - 28688.05 = €\,65{,}508.45$

Extra investment to amount to this

$P = 65508.45/(1.04)^4 = €55996.90$

10. An air-conditioning system cost €8000. A straight line depreciation and a reducing balance curve for this system are shown below.



(i) Using the graph, estimate the rate of depreciation.
(Let the value after 20 years be €1.)
(ii) Explain why the reducing balance curve can never have a zero value.
(iii) Find the slope of the straight line representing depreciation.
(iv) Estimate the point of intersection of the two graphs.
(v) After 5 years, what is the value of the system on a reducing balance basis?
(iv) In your opinion, which method of depreciation gives a more realistic value for the system? Explain your answer.

$F=P(1-r)^t$

$1-r=\sqrt[t]{\dfrac{F}{P}}$

(i) $t=20 \quad P=€8000 \quad F=€1 \quad r=?$

$1-r = \sqrt[20]{\dfrac{1}{8000}} = 0.638 \Rightarrow r = 1-0.638 \simeq 0.36$

$\simeq 36\%$

(ii) Because the function approaches a limit. The value is always being reduced by 36%, a positive number being multiplied by 0.64 is > 0.

(iii) Slope is negative

$|m| = \frac{Rise}{Run} = \frac{8000}{5} = 1600$

$\Rightarrow m = -1600$

$F = P(1-r)^t$ (iv) $t = 5$, $P = €8000$, $r = 20\%$

$F = ?$

$F = 8000(0.64)^5 \simeq €859$

(v) The reducing balance is more realistic because the AC system should still have a value in 5 years.

Section 5.3

In this example we sum a geometric series.

Example 1

Catríona saves €400 every three months for five years at an effective quarterly rate of 0.9%.

(i) Represent her savings by a geometric series

(ii) Find the value of her investment at the end of the period.

$i = 0.9\% = 0.009$

5 years = (5×4) quarters = 20 payments

(i) Catríona's savings are represented by

$400(1.009) + 400(1.009)^2 + 400(1.009)^3 + \ldots\ldots 400(1.009)^{20}$

(ii) $a = 400(1.009)$, $r = (1 + i) = 1.009$, $n = 20$

$$S_n = \frac{a(1 - r^n)}{(1 - r)}$$

$$S_n = \frac{400(1.009)[1 - (1.009)^{20}]}{1 - 1.009}$$

$= €8800.89$

a = first term

Exercise 5.3

1. Calculate the future value of 36 monthly instalments of €20.00 at an interest rate of 0.5% per month. What is the total interest earned on these savings?

$A = P(1+i)^t$

MER $= 0.5\% = 0.005$

geometric Series $= \underbrace{20(1.005)^{36}}_{T_{36}} + \underbrace{20(1.005)^{35}}_{T_{35}} + \dots + \underbrace{20(1.005)^1}_{T_1}$

$S_n = \dfrac{a(1-r^n)}{1-r}$

$T_1 = a = 20(1.005)^1 = 20.1$

$r = 1.005$ (Common Ratio)

$n = 36$

$S_{36} = \dfrac{20.1(1 - 1.005^{36})}{1 - 1.005}$

$= €790.66$

2. Marie has saved €30.00 per month since her 18th birthday.
If her bank has guaranteed her an interest rate of 4% per annum, find

(i) the equivalent monthly rate of interest, correct to two places of decimals
(ii) the value of her savings on her 21st birthday.

(i) AER $= 4\% = i$ MER $= ? = r$

$(1+r)^{12} = (1+i)^1$

$(1+r)^{12} = (1+i)^1 \Rightarrow r = \sqrt[12]{1+i} - 1$

MER $= \sqrt[12]{1.04} - 1 = 0.00327$

time? terms? $\Rightarrow n = 36$

(ii) 18th → 21st birthday = 3 years = 36 months

geometric series $= \underbrace{30(1.00327)^{36}}_{T_{36}} + \underbrace{30(1.00327)^{35}}_{T_{35}} + \dots + \underbrace{30(1.00327)}_{T_1 = a}$

$n = 36$, $a = 30(1.00327)^1 \approx 30.1$, ratio $r = 1.003$

$S_n = \dfrac{a(1-r^n)}{1-r}$

$S_{36} = \dfrac{30.1(1 - 1.00327^{36})}{1 - 1.00327} \approx €1148$