3. A special savings account offers an AER of 4% per annum. If I invest €2000 per year in this account, how much will my investment be worth in 5 years time?

$F = P(1+i)^t$

Geometric Series

$$2000(1.04)^5 + 2000(1.04)^4 + \dots + 2000(1.04)^1$$
$$T_5 + T_4 + \dots + T_1$$

$S_n = \frac{a(1-R^n)}{1-R}$

$a = T_1 = 2000(1.04)^1 = 2080$

Ratio: $R = 1.04$

$n = 5$

$$S_5 = \frac{2080(1-1.04^5)}{1-1.04} =$$

$$= €11,265.95$$

4. Show that the future value of a series of n payments of €P, earning an interest rate of i% per annum, can be written as:

$$\text{Future value} = P(1+i)\left(\frac{(1+i)^n - 1}{i}\right)$$

$F = P(1+i)^t$

assume n payments means 1 per year for n years. ?

Geometric Series

$$P(1+i)^n + P(1+i)^{n-1} + \dots + P(1+i)^1$$
$$T_n \qquad T_{n-1} \qquad T_1$$

$S_n = \frac{a(1-R^n)}{1-R}$

$a = T_1 = P(1+i)$

$R = 1+i$

$n = n$

$$S_n = \frac{P(1+i)(1-(1+i)^n)}{1-(1-i)}$$

$$= P(1+i)\frac{(1-(1+i)^n)}{\cancel{1}-\cancel{1}-i}$$

multiply above and below by -1

$$= P(1+i)\left(\frac{(1+i)^n - 1}{i}\right)$$

6. Anne received a cheque in the post for €6523.33 after saving for 5 years with her bank in a scheme offering 9% per annum. If she invested €A per annum,
   (i) write down a geometric series representing the value of her investment over the 5 years
   (ii) find the value of A.

$F = P(1+i)^t$

Geometric Series

$$A(1.09)^5 + A(1.09)^4 + \dots + A(1.09)^1$$

T_5 T_4 T_1

$S_n = \frac{a(1-R^n)}{1-R}$

$a = T_1 = A(1.09)$

Ratio: $R = 1.09$

$n = 5$

$S_5 = €6523.33$

$S_5 =$

$$6523.33 = \frac{A(1.09)(1-1.09^5)}{1-1.09}$$

$\div 6.52333$

$$6523.33 = A(6.52333)$$

$$A = €1000$$

7. Use the future value formula to find the final value if €200 is invested every month for 2 years. The interest rate is 9% per annum, compounded monthly.

change annual equivalent rate to monthly equivalent rate

9% p.a. $\Rightarrow$ MER ?

$(1.09)^1 = (1+R)^{12} \Rightarrow R = \sqrt[12]{1.09} - 1$

$R = 0.0072$

Instalments?

2 years $= (12)2 = 24$ months

$F = P(1+i)^t$

Geometric Series

$$200(1.0072)^{24} + 200(1.0072)^{23} + \dots + 200(1.0072)^1$$

T_{24} T_{23} T_1

$S_n = \frac{a(1-R^n)}{1-R}$

$a = T_1 = 200(1.0072)^1 = 201.44$

Ratio: $R = 1.0072$

$n = 24$

Future Value =

$$S_{24} = \frac{201.44(1-1.0072^{24})}{1-1.0072}$$

$$= €5256.82$$

8. George wants to make regular payments into an account that pays 8.5% per annum compound interest in order to have €10 000 after 7 years. Find the amount of each annual payment.

$F = P(1+i)^t$

Geometric Series

Principle for each month is unknown $= P$

$$\underbrace{P(1\cdot085)^7}_{T_7} + \underbrace{P(1\cdot085)^6}_{T_6} + \ldots + \underbrace{P(1\cdot085)^1}_{T_1}$$

$a = T_1 = P(1\cdot085)$

Ratio $= 1\cdot085$

$n = 7$

$S_7 = €\ 10\,000$

$$S_n = \frac{a(1-R^n)}{1-R}$$

$$S_7 \Rightarrow \quad 10\,000 = \frac{P(1\cdot085)(1-1\cdot085^7)}{1-1\cdot085}$$

$$10\,000 = P(9\cdot8306)$$

$$P = 10000 / 9\cdot8306$$

$$= €\ 1017\cdot22 \checkmark$$

9. Ella wants to have €5000 in 3 years time.
She invests in an annuity that pays 7.2% per annum, compounded quarterly.
How much does she need to deposit each quarter to achieve her target of €5000?

$(1+i)^1 = (1+R)^4$

7·2% p.a $\Rightarrow$ quarterly equivalent Rate? (R)

QER?

$(1\cdot072)^1 = (1+R)^4 \Rightarrow R = \sqrt[4]{1\cdot072} - 1 \quad = 0\cdot0175$

number of instalments?

3 years $= 3(4) = 12$ quarters

$F = P(1+i)^t$

Let each instalment = P

Geometric Series

$$\underbrace{P(1\cdot0175)^{12}}_{T_{12}} + \underbrace{P(1\cdot0175)^{11}}_{T_{11}} + \ldots + \underbrace{P(1\cdot0175)^1}_{T_1}$$

$T_1 = a = P(1\cdot0175)$

Ratio $= R = 1\cdot0175$

$n = 12$

$S_{12} = 5000$

$$S_n = \frac{a(1-R^n)}{1-R}$$

$$S_{12} \Rightarrow \quad 5000 = \frac{P(1\cdot0175)(1-1\cdot0175^{12})}{1-1\cdot0175} = P(13\cdot456$$

$$P = 5000/13\cdot4565 = €371\cdot57 \checkmark$$

10. Prove that the present value of an annuity (instalments paid at the beginning of each period) is given by:
Future value (calculated at the end of each period) ÷ $(1 + i)^n$.

To Prove $P = \dfrac{F}{(1+i)^n}$

If P is invested at a rate of i for 1 period of time

1 period $\Rightarrow F = P(1+i)$

2 periods $\Rightarrow F = P(1+i)(1+i) = P(1+i)^2$

3 periods $\Rightarrow F = P(1+i)(1+i)(1+i) = P(1+i)^3$

n periods $\Rightarrow F = P(1+i)^n$

$\Rightarrow P = \dfrac{F}{(1+i)^n}$

11. Show how the present value of an annuity involving depositing €3000 per year in an account for 6 years can be written as a geometric series, given that the interest rate is 8% per annum.
(i) Calculate the present value.
(ii) Calculate the future value of the annuity.
(iii) If the present value of the annuity in (i) was put on deposit as a single investment at 8% per annum, show that it will amount to the same future value of the annuity after 6 years.

(i)

The first instalment is €3000 this is needed immediately and won't be earning interest

$P = \dfrac{F}{(1+i)^t}$

Geometric Series

$$\underbrace{\frac{3000}{(1\cdot08)^5}}_{T_6} + \underbrace{\frac{3000}{(1\cdot08)^4}}_{T_5} + \dots \underbrace{\frac{3000}{(1\cdot08)^1}}_{T_2} + \underbrace{3000}_{T_1}$$

$S_n = \dfrac{a(1-R^n)}{1-R}$

$T_1 = a = 3000$

$R = 1/1\cdot08$

$n = 6$

Present Value = $S_6 = \dfrac{3000\left(1-(1/1\cdot08)^6\right)}{1-(1/1\cdot08)}$

$= €14{,}978\cdot13$ ✓

11. Show how the present value of an annuity involving depositing €3000 per year in an account for 6 years can be written as a geometric series, given that the interest rate is 8% per annum.
(i) Calculate the present value.
(ii) Calculate the future value of the annuity.
(iii) If the present value of the annuity in (i) was put on deposit as a single investment at 8% per annum, show that it will amount to the same future value of the annuity after 6 years.

(ii)

$F = P(1+i)^t$

Geometric Series

$3000(1\cdot08)^6 + 3000(1\cdot08)^5 + \ldots + 3000(1\cdot08)^1$

T_6 T_5 .... T_1

$a = T_1 = 3000(1\cdot08) = 3240$

Ratio, $R = 1\cdot08$

$n = 6$

$S_n = \dfrac{a(1-R^n)}{1-R}$

$S_6 = \dfrac{3240(1-1\cdot08^6)}{1-1\cdot08} = €\ 23{,}768\cdot41$ ✓

11. Show how the present value of an annuity involving depositing €3000 per year in an account for 6 years can be written as a geometric series, given that the interest rate is 8% per annum.
(i) Calculate the present value.
(ii) Calculate the future value of the annuity.
(iii) If the present value of the annuity in (i) was put on deposit as a single investment at 8% per annum, show that it will amount to the same future value of the annuity after 6 years.

(iii)

$F = P(1+i)^t$

$P = €14{,}978\cdot13$
$i = 8\%$
$t = 6$ years

$F = 14978\cdot13(1\cdot08)^6$

$= €\ 23{,}768\cdot20$ ✓