Summary

The future value of
n payments of €P at i%

$$\text{Future value} = P(1+i)\left(\frac{1-(1+i)^n}{1-(1+i)}\right)$$

$$= P(1+i)\left(\frac{(1+i)^n - 1}{i}\right)$$

The present value (cost) of
n payments of €P at i%

$$\text{Present value} = \left(\frac{P}{1+i}\right)\left[\frac{1-\left(\frac{1}{1+i}\right)^n}{1-\left(\frac{1}{1+i}\right)}\right]$$

$$= \frac{P}{(1+i)^n}\left(\frac{(1+i)^n - 1}{i}\right)$$

Section 5.4 Loans – Mortgages

If we wish to calculate the repayments needed for a car loan or a mortgage on a house, we use the same procedure for finding the present value as was used in the previous section.

The sum of the present values of each repayment over the given period of time must be equal to the value of the car loan or mortgage.

$$\text{€ Mortgage} = \frac{\text{€ Payment}}{1+i} + \frac{\text{€ Payment}}{(1+i)^2} + \frac{\text{€ Payment}}{(1+i)^3} + \dots \frac{\text{€ Payment}}{(1+i)^n}$$

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$$= \left(\frac{\text{€ Payment}}{1+i}\right)\left[\frac{1-\left(\frac{1}{1+i}\right)^n}{1-\left(\frac{1}{1+i}\right)}\right]$$

$$\therefore \quad \text{€ Payment} = \frac{\text{€ Mortgage } (i)(1+i)^n}{(1+i)^n - 1}$$

(*formulae and tables,* p.31)

i = the effective monthly rate of interest (expressed as a decimal)
n = the number of payments (years/months)
€M = the amount of the mortgage or loan
€P = the repayment per month

Example 1

Calculate the size of the monthly repayments needed for a car loan of €10 000 if the loan is to be repaid over a 5-year term at an effective monthly rate of 0.72%.

$$A = \frac{P(i)(1+i)^n}{(1+i)^n - 1}$$

5 yrs = 60 months

$$A = \frac{10000\,(\cdot 0072)(1\cdot 0072)^{60}}{1\cdot 0072^{60} - 1}$$

$$= €206$$

$$\therefore \quad € \text{ Payment} = \frac{€ \text{ Mortgage } (i)(1+i)^n}{(1+i)^n - 1}$$

(*formulae and tables,* p.31)

Example 2

Find the monthly repayments required for a mortgage of €150 000, based on an annual rate of 4.5% over 20 years.

$$(1+i)^1 = (1+R)^{12}$$

$$\text{MER } R = \sqrt[12]{1+i} - 1$$

$$R = \sqrt[12]{1\cdot 045} - 1 = 3\cdot 6748 \times 10^{-3}$$
$$= 0\cdot 00367$$

$$A = \frac{P(i)(1+i)^t}{(1+i)^t - 1}$$

20 years = 240 months

$$\text{Payments} = \frac{150\,000\,(0\cdot 00367)(1\cdot 00367)^{240}}{1\cdot 00367^{240} - 1}$$

$$= 941\cdot 22$$

$$\therefore \quad € \text{ Payment} = \frac{€ \text{ Mortgage } (i)(1+i)^n}{(1+i)^n - 1}$$

(*formulae and tables,* p.31)

Exercise 5.4

1. Calculate the monthly repayments required for a mortgage of €200 000, paid over a 30-year period at an annual interest rate of 6%.

$$A = \frac{P(i)(1+i)^t}{(1+i)^t - 1}$$

30 years = 360 months

$\therefore$ € Payment $= \dfrac{\text{€ Mortgage } (i)(1+i)^n}{(1+i)^n - 1}$

(*formulae and tables*, p.31)

MER = $R = \left(\sqrt[12]{1.06}\right) - 1 = 0.00487$

$$\text{Payments} = \frac{200000(0.00487)(1.00487)^{360}}{1.00487^{360} - 1}$$

$\simeq$ €1179

2. Alice wants to take out a 20-year mortgage.
The average interest rate over the lifetime of the mortgage is 8% per annum.
Alice can afford repayments of €850 per month.
What is the largest mortgage she can afford?
Give your answer to the nearest €100.

$\therefore$ € Payment $= \dfrac{\text{€ Mortgage } (i)(1+i)^n}{(1+i)^n - 1}$

(*formulae and tables*, p.31)

Mortgage = ?

MER ?

$R = \sqrt[12]{1.08} - 1 = 6.434 \times 10^{-3} = 0.006434$

$$A = \frac{P(i)(1+i)^t}{(1+i)^t - 1}$$

$$\Rightarrow P = A\frac{((1+i)^t - 1)}{(i)(1+i)^t}$$

20 years = 240 months

$$\Rightarrow \text{mortgage} = \frac{850(1.006434^{240} - 1)}{(0.006434)(1.00634^{240})}$$

$\approx$ €104,700