#### **Summary**

The future value of n payments of €P at i%

Future value = 
$$P(1 + i) \left( \frac{1 - (1 + i)^n}{1 - (1 + i)} \right)$$
  
=  $P(1 + i) \left( \frac{(1 + i)^n - 1}{i} \right)$ 

The present value (cost) of n payments of  $\in P$  at i%

Present value 
$$= \left(\frac{P}{1+i}\right) \left[\frac{1 - \left(\frac{1}{1+i}\right)^n}{1 - \left(\frac{1}{1+i}\right)}\right]$$
$$= \frac{P}{(1+i)^n} \left(\frac{(1+i)^n - 1}{i}\right)$$

## Section 5.4 Loans – Mortgages -

If we wish to calculate the repayments needed for a car loan or a mortgage on a house, we use the same procedure for finding the present value as was used in the previous section.

The sum of the present values of each repayment over the given period of time must be equal to the value of the car loan or mortgage.

$$\mathcal{E} \text{ Mortgage} = \frac{\mathcal{E} \text{ Payment}}{1+i} + \frac{\mathcal{E} \text{ Payment}}{(1+i)^2} + \frac{\mathcal{E} \text{ Payment}}{(1+i)^3} + \dots \frac{\mathcal{E} \text{ Payment}}{(1+i)^n}$$

$$= \left(\frac{\mathcal{E} \text{ Payment}}{1+i}\right) \left[\frac{1-\left(\frac{1}{1+i}\right)^n}{1-\left(\frac{1}{1+i}\right)}\right]$$

$$\therefore \quad \in \text{Payment} = \frac{\text{ } \in \text{Mortgage } (i)(1+i)^n}{(1+i)^n - 1}$$

(formulae and tables, p.31)

i = the effective monthly rate of interest (expressed as a decimal)
n = the number of payments (years/months)
€ M = the amount of the mortgage or loan
€ P = the repayment per month

### **Example 1**

Calculate the size of the monthly repayments needed for a car loan of €10 000 if the loan is to be repaid over a 5-year term at an effective monthly rate of 0.72%.

$$A = \frac{\int (i)(1+i)^n}{(1+i)^n - 1}$$

$$\therefore \quad \in \text{Payment} = \frac{\in \text{Mortgage } (i)(1+i)^n}{(1+i)^n - 1}$$

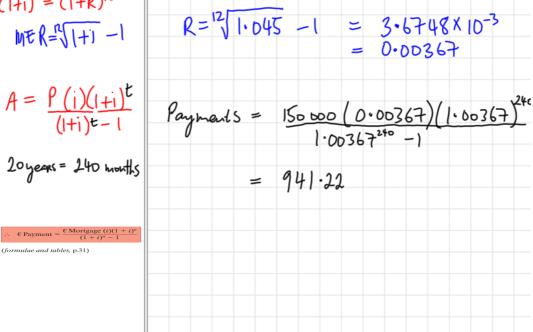
# **Example 2**

Find the monthly repayments required for a mortgage of €150 000, based on an annual rate of 4.5% over 20 years.

$$(1+i)^{1} = (1+R)^{12}$$

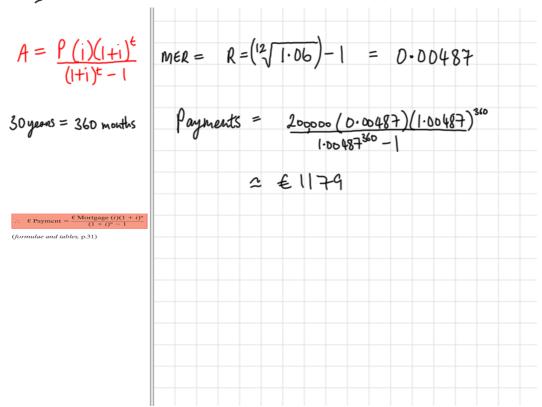
$$A = \frac{\rho(i)(1+i)^t}{(1+i)^t-1}$$





#### **Exercise 5.4**

1. Calculate the monthly repayments required for a mortgage of €200 000, paid over a 30-year period at an annual interest rate of 6%.



2. Alice wants to take out a 20-year mortgage.

The average interest rate over the lifetime of the mortgage is 8% per annum.

Alice can afford repayments of €850 per month.

What is the largest mortgage she can afford?

