

SEC SET E: PAPER 2

QUESTION 5 (25 MARKS)

Question 5 (a)

Solve the equation $\cos 3\theta = \frac{1}{2}$, for $\theta \in \mathbb{R}$, (where θ is in radians).

SOLUTION

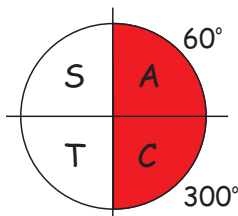
$\cos 3\theta = +\frac{1}{2}$ [The cos function is positive meaning the answer can occur in the first and fourth quadrants.]

$$\therefore 3\theta = \cos^{-1}\left(\frac{1}{2}\right) = 60^\circ = \frac{\pi}{3}$$

FIRST QUADRANT

$$3\theta = \frac{\pi}{3} + 2n\pi$$

$$\theta = \frac{\pi}{9} + \frac{2n\pi}{3}$$



The answer will repeat every 360° or 2π times.

FOURTH QUADRANT

$$3\theta = \frac{5\pi}{3} + 2n\pi$$

$$\theta = \frac{5\pi}{9} + \frac{2n\pi}{3}$$

Question 5 (b & c)

(b) The graphs of three functions are shown on the diagram below. The scales on the axes are not labelled. The three functions are:

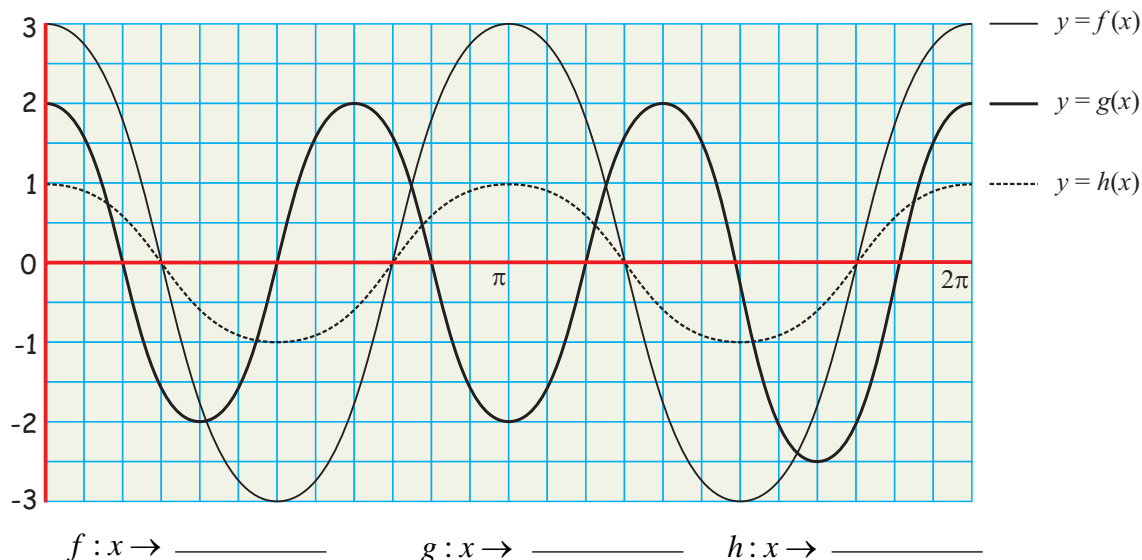
$$x \rightarrow \cos 3x$$

$$x \rightarrow 2 \cos 3x$$

$$x \rightarrow 3 \cos 2x$$

Identify which function is which, and write your answers in the spaces below the diagram.

(c) Label the scales on the axes in the diagram in part (b).



SOLUTION

$$x \rightarrow \cos 3x: R = [-1, 1], P = \frac{2\pi}{3}$$

$$x \rightarrow 2 \cos 3x: R = [-2, 2], P = \frac{2\pi}{3}$$

$$x \rightarrow 3 \cos 2x: R = [-3, 3], P = \frac{2\pi}{2} = \pi$$

$$y = a \cos nx$$

$$R = [-a, a], P = \frac{2\pi}{n}$$

$$h: x \rightarrow \cos 3x$$

$$g: x \rightarrow 2 \cos 3x$$

$$f: x \rightarrow 3 \cos 2x$$