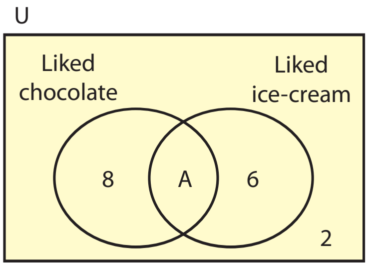
7. In a class of 40 children, a survey was carried out to find out how many children liked chocolate and how many liked ice-cream. The Venn diagram shows the results but the region marked A is not filled in.



(i) What is the number in region A?
(ii) What can you say about the children in region A?
(iii) If one child is chosen at random, what is the probability that the child liked ice-cream but not chocolate?
(iv) One of the children who liked chocolate is chosen at random. What is the probability that the child also liked ice-cream?

SECTION 6.7 The multiplication rule – Bernoulli trials

We show below the sample space for tossing a coin and throwing a dice.

Dice

Coin	1	2	3	4	5	6
H	H1	H2	H3	H4	H5	H6
T	T1	T2	T3	T4	T5	T6

From the sample space, we can see that

$P(\text{H}, 6) = \frac{1}{12}$.

Whether or not the coin lands on 'head' has no effect on whether the dice shows a 6 or any other score. The two events, 'getting a head' and 'scoring a 6', are independent.

Now the probability of getting a head when tossing a coin is $\frac{1}{2}$.

The probability of getting a 2 on the dice is $\frac{1}{6}$.

If we multiply the two probabilities $\frac{1}{2}$ and $\frac{1}{6}$ we get $\frac{1}{12}$, as found above.

This illustrates the **multiplication rule** of probability which is given below.

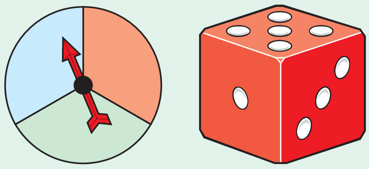
Multiplication Rule: $P(\text{A and B}) = P(\text{A}) \times P(\text{B})$

The multiplication rule is particularly useful when dealing with two or more events where each event is independent of the other. It provides an alternative approach to problems such as throwing two dice, already dealt with in Section 6.3.

The multiplication rule is generally referred to as the **AND** rule.

Example 1

Amanda throws an ordinary dice and spins the spinner shown. Each colour is equally likely. Find the probability that she gets a red and an even number.



$P(\text{red}) = \frac{1}{3}$ $\quad$ $P(\text{even number}) = \frac{3}{6} = \frac{1}{2}$

$\therefore \quad P(\text{red and even number}) = \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$

Note: You can also get this result by using a sample space.

Example 2

Mary and John have their birthdays in the same week.
Find the probability that
(i) Mary's birthday falls on Monday
(ii) both have their birthdays on Monday
(iii) both have their birthdays on either Saturday or Sunday.

(i) $P(\text{Mary's birthday on Monday}) = \frac{1}{7}$

(ii) $P(\text{both birthdays on Monday}) = P(M, \text{Mon}) \times P(J, \text{Mon})$
$= \frac{1}{7} \times \frac{1}{7} = \frac{1}{49}$

(iii) $\text{P(both birthdays on Sat or Sun)} = \frac{2}{7} \times \frac{2}{7}$
$= \frac{4}{49}$

Bernoulli trials

Consider the experiment of throwing a dice and requiring a 6 to start a game.

If a 6 is thrown it can be regarded as a 'success'. Any other number thrown is a 'failure'.

If each throw of the dice is regarded as a **trial**, then

- for each trial there are two possible outcomes, 'success' and 'failure'
- the probability of success (getting a 6) is the same for each trial
- each trial is independent of the outcomes of other trials.

When an experiment consists of repeated trials and the conditions listed above exist, such trials are known as **Bernoulli trials**, named after James Bernoulli.

In the experiment above, if a 6 is thrown for the first time on the third trial, we say that 'the first success occurs on the third trial'.

James Bernoulli (1654–1705) was a Swiss mathematician who did pioneering work in probability and calculus.

For our course, we will deal with problems that involve up to three Bernoulli trials only.

Example 3

A fair coin is tossed until a head occurs.
Find the probability that the first head occurs on the third toss.

If the first head occurs on the 3rd toss, then the first two tosses show tails, i.e., TTH

$P(\text{H}) = \frac{1}{2}$ and $P(\text{T}) = \frac{1}{2}$

$P(\text{TTH}) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

$\therefore$ the probability that a head occurs on the 3rd toss is $\frac{1}{8}$.

> T = tail
> H = head

Example 4

A fair dice is rolled repeatedly.
Find the probability that a 5 or a 6 first appears on the third throw.

Let a 5 or a 6 represent 'success' (S) and 1, 2, 3, 4 represent 'failure' (F).

$P(\text{success}) = \frac{2}{6} = \frac{1}{3}$ and $P(\text{failure}) = \frac{4}{6} = \frac{2}{3}$

If the first 'success' is on the 3rd throw, the sequence is FFS.

$P(\text{FFS}) = \frac{2}{3} \times \frac{2}{3} \times \frac{1}{3} = \frac{4}{27}$

$\therefore$ probability that 5 or 6 first appears on 3rd throw is $\frac{4}{27}$.

Example 5

A candidate takes a 3-question multiple choice test.
There are four choices in each question.
If she guesses on each question, what is the probability that
(i) she gets all three answers correct
(ii) she gets the first two answers wrong but the third correct?

(i) There are 4 choices;
so $P(\text{correct answer}) = \frac{1}{4}$ and $P(\text{wrong answer}) = \frac{3}{4}$.

$P(\text{all three correct}) = \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} = \frac{1}{64}$

(ii) $P(\text{first 2 wrong and 3rd correct}) = P(\text{wrong}) \times P(\text{wrong}) \times P(\text{correct})$
$= \frac{3}{4} \times \frac{3}{4} \times \frac{1}{4} = \frac{9}{64}$

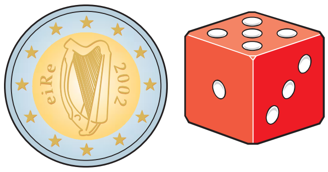
Note: If S stands for 'success' and F stands for 'failure' in Bernoulli trials, then the probability of only one 'success' in three trials is the sum of these three probabilities:

$P(\text{FFS}) + P(\text{FSF}) + P(\text{SFF})$

Exercise 6.7

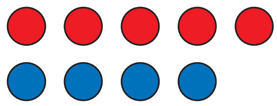
1. A coin is tossed twice. What is the probability of getting
   (i) 2 heads
   (ii) a head on the first toss and a tail on the second?

2. A coin is tossed and a dice is thrown.
   What is the probability of getting
   (i) a head and a 6
   (ii) a tail and an even number
   (iii) a head and a multiple of 3?



3. Two dice are thrown. Find the probability of getting
   (i) 2 fives
   (ii) 2 even numbers
   (iii) both dice showing a number less than 3.

4. A bag contains 5 red discs and 4 blue discs. A disc is selected at random and then replaced. A second disc is then selected.
   Find the probability that
   (i) both discs are red
   (ii) the first is red and the second is blue
   (iii) the first is blue and the second is red.

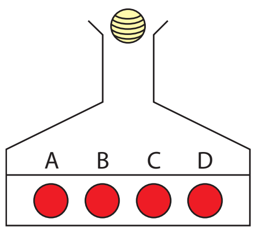


5. In an experiment, a card is drawn from a pack of 52 and a dice is thrown.
   Find the probability of obtaining
   (i) a diamond and a 6 on the dice
   (ii) a black card and an even number on the dice
   (iii) a heart and a multiple of 3 on the dice.

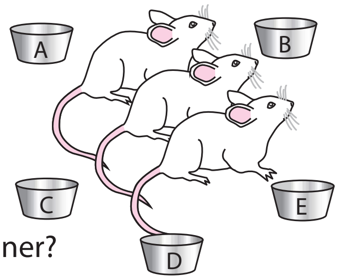
There are 13 diamonds, 26 black cards and 13 hearts in a pack of playing cards.

6. The letters of the word ALGEBRA are written on individual cards and the cards are then put into a box. A card is selected at random and then replaced. A second card is then selected.
   Find the probability of obtaining
   (i) the letter A twice
   (ii) the letters G and E in that order
   (iii) the letter R twice
   (iv) two vowels.

7. When a tennis ball is dropped into the device shown, it is equally likely to come out any of the holes marked A, B, C and D.
   If a tennis ball is dropped in, find the probability that it will come out of the hole marked
   (i) A
   (ii) A or C.
   If two other balls are dropped in one after the other, find the probability that
   (iii) both will come out the hole marked A
   (iv) both will come out the hole marked A or both will come out the hole marked C.



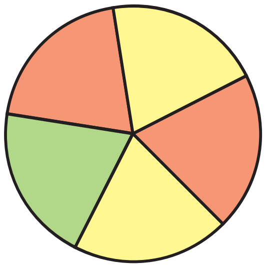
8. There are three tame mice, Sam, Pam and Ham in an enclosure.
They can choose to eat at five containers (A, B, C, D and E).
The choice is totally random.



(i) What is the probability that Sam will eat from container A?

(ii) What is the probability that Sam and Pam will both eat
(a) from container A (b) from the same container?

9. This spinner is spun twice. Each sector is equally likely.
Find the probability that



(i) the first colour is yellow
(ii) the first two colours are red and green in that order
(iii) the first two colours are red
(iv) the first two colours are both red or both yellow.

10. Ann and Barry celebrate their birthdays in a particular week.
Assuming that the birthdays are equally likely to fall on any day of the week, what is the probability that
(i) Ann's birthday is on Wednesday
(ii) both birthdays are on Monday
(iii) both birthdays are on a day beginning with T?

11. James tosses a fair coin several times.
Find the probability for each of these events:
(i) the first head occurs on the second toss
(ii) the first head occurs on the third toss.

12. Katie throws a fair dice until she gets a 6.
Calculate the probability that she gets
(i) 6 on the first throw
(ii) the first 6 on the third throw.

13. The probability of getting a head with a biased coin is $\frac{2}{3}$.
Jack tosses the coin three times.
(i) Calculate the probability that he gets heads on all three throws.
(ii) Calculate the probability that he gets the first head on
(a) the second throw (b) the third throw.

14. 25% of pupils in a school travel to school by bus.
Three pupils in the school are selected at random.
Find the probability that
(i) all three travel by bus
(ii) the first two pupils selected do not travel by bus
(iii) the first two pupils do not travel by bus but the third one does.

15. A bag contains 3 red beads and 2 green beads. A bead is selected from the bag and then replaced. This process is repeated.
Find the probability that
(i) the first bead selected is green
(ii) the first green bead selected is at the third attempt.

16. Andy plays a series of tennis matches against the same opponent.
The probability that he wins any match is $\frac{4}{5}$.
Calculate the probability that
(i) Andy has his first win in his second match
(ii) Andy has his first win in his third match
(iii) Andy loses all three matches.

17. The probability that it will rain on any given day in May is 0.3.
If three days in May are selected at random, find the probability that
(i) the first day has no rain
(ii) the first two days will have rain
(iii) the third day is the first day to have rain.

18. A cube has the letter 'A' on four faces and the letter 'B' on the remaining two faces.
It is thrown three times.
Calculate the probability of obtaining
(i) B on the first throw
(ii) the first B on the second throw
(iii) the first A on the third throw.

19. Shane draws a card from a normal pack of playing cards and then replaces it.
He does this three times.
Calculate the probability that he selects
(i) a diamond on the first draw
(ii) the first diamond on the third draw
(iii) only one diamond in the three draws.

20. A fair coin is tossed at the start of each match in a 3-match series.
One captain tosses and the other calls 'Heads' or 'Tails'.
Find the probability that the toss is called correctly
(i) three times
(ii) exactly once
(iii) exactly twice.

21. Andy is playing football and tennis.
He has one match in each sport to play.
The probability that he wins the football match is 0.3.
The probability that he will draw is 0.5.

He has a 0.6 chance of winning the tennis match, otherwise he will lose.
Find the probability that
(i) Andy loses the football match
(ii) Andy wins both matches
(iii) Andy loses both matches
(iv) Andy wins the football match and loses the tennis match.

22. In a board game, a counter is moved along the squares by an amount equal to the number thrown on a fair dice. If you land on a square at the bottom of a ladder you move the counter to the square at the top of that ladder.

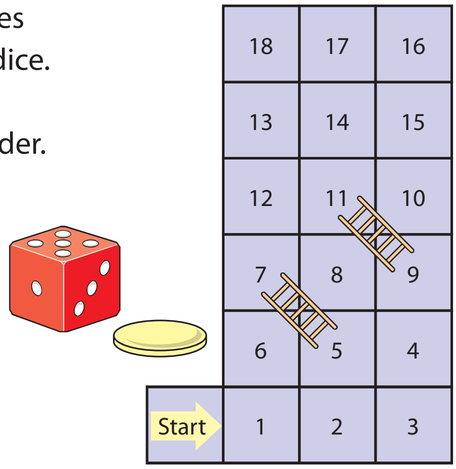
(i) What is the probability that a player reaches square 4 with one throw of the dice?

(ii) What is the probability that a player can reach square 7 with one throw of the dice?

(iii) What is the probability of taking two throws to get to square 2?

(iv) List the three possible ways to land on square 18 with exactly three throws of the dice.

(v) Calculate the probability of landing on square 18 with exactly three throws of the dice.



Section 6.8 Tree diagrams

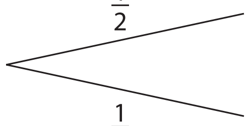
The possible outcomes of two or more events can be shown in a particular type of diagram called a **tree diagram**.

In a tree diagram

(i) write the outcomes at the end of each branch

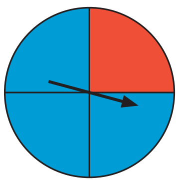
(ii) write the probabilities on each branch.

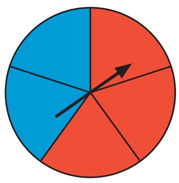
In this tree diagram representing the outcomes when a coin is tossed, there are two branches (or two outcomes).



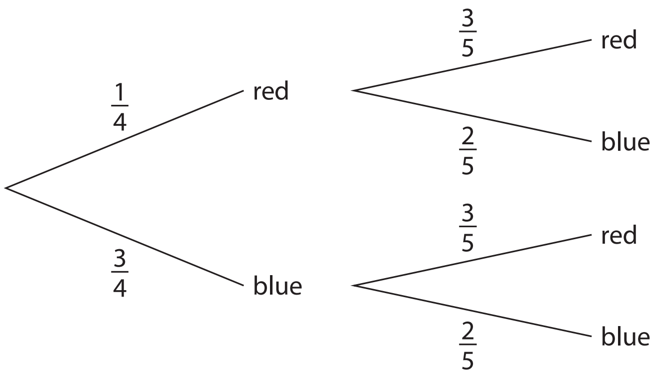
The probability, $\frac{1}{2}$, is written on each branch.

The possible outcomes when these two spinners are spun can be shown in a tree diagram.


Spinner A


Spinner B

Probabilities along the branches are found by multiplying.



probability of (A red, B red) $= \frac{1}{4} \times \frac{3}{5} = \frac{3}{20}$

probability of (A red, B blue) $= \frac{1}{4} \times \frac{2}{5} = \frac{2}{20}$

probability of (A blue, B red) $= \frac{3}{4} \times \frac{3}{5} = \frac{9}{20}$

probability of (A blue, B blue) $= \frac{3}{4} \times \frac{2}{5} = \frac{6}{20}$