

Integration  
 Anti-Differentiation  
 Tutorial Questions



Tutorial videos

**Integration**

Constants of integration omitted.

$f(x)$	$\int f(x)dx$
$x^n, (n \neq -1)$	$\frac{x^{n+1}}{n+1}$
$\frac{1}{x}$	$\ln x $
$e^x$	$e^x$
$e^{ax}$	$\frac{1}{a}e^{ax}$
$a^x (a > 0)$	$\frac{a^x}{\ln a}$
$\cos x$	$\sin x$
$\sin x$	$-\cos x$
$\tan x$	$\ln \sec x $
$\frac{1}{\sqrt{a^2 - x^2}} (a > 0)$	$\sin^{-1} \frac{x}{a}$
$\frac{1}{x^2 + a^2} (a > 0)$	$\frac{1}{a} \tan^{-1} \frac{x}{a}$

**Example 1**

Find (i)  $\int(3x^2 + 4x + 5) dx$  (ii)  $\int(2x - 1)^2 dx$ .

## Example 2

Find (i)  $\int \frac{x^3 - 4x}{x} dx$  (ii)  $\int \left( x^3 + \frac{1}{x^2} + \sqrt{x} \right) dx$  (iii)  $\int \sqrt{x}(x + 4) dx$

## Example 3

A curve with equation  $y = f(x)$  passes through the point  $(2, 0)$ .

If  $f'(x) = 3x^2 - \frac{1}{x^2}$ , find  $f(x)$ .

### Example 1

Find the antiderivative of each of the following:

(i)  $\int e^{3x} dx$

(ii)  $\int (e^{4x} + 6x) dx$

(iii)  $\int (e^{5x} + 2) dx$

(iv)  $\int (e^x + e^{-x}) dx$

### Example 2

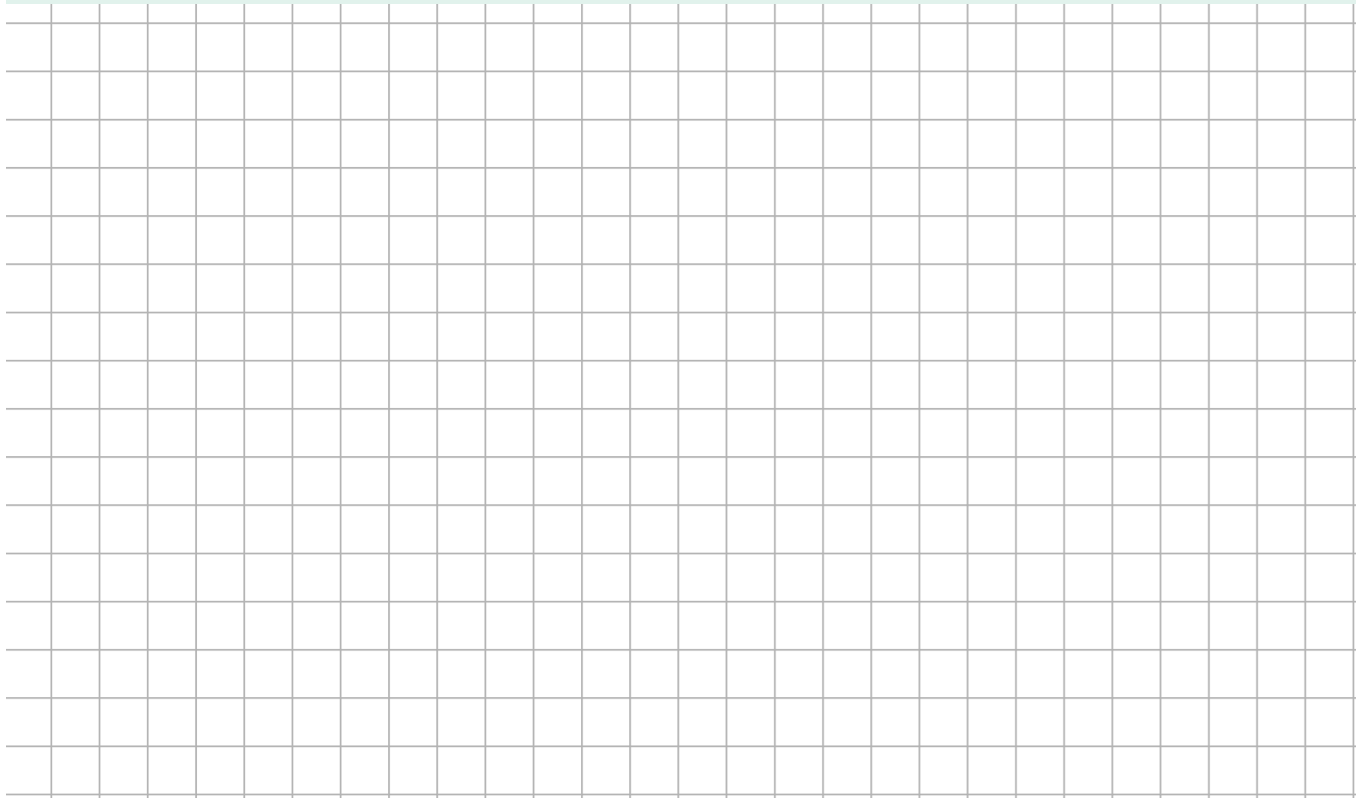
Given  $y = 5^x$ , use the rules of logarithms to find  $x$  in terms of  $y$ .

Hence, find (i)  $\frac{dx}{dy}$  (ii)  $\frac{dy}{dx}$ .

Use the result from (ii) to show that  $\int 5^x dx = \frac{5^x}{\ln 5} + c$ .

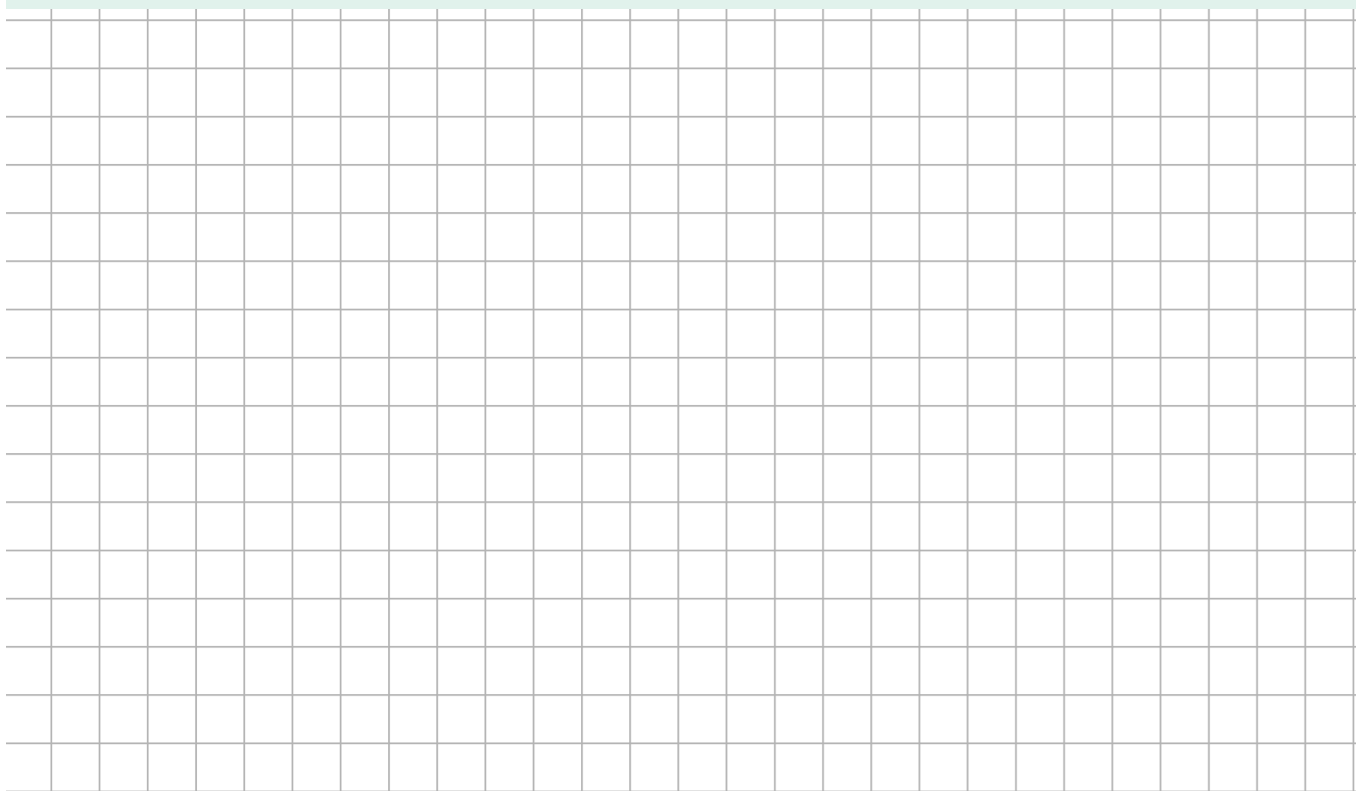
### Example 3

Find (i)  $\int \cos 4x \, dx$       (ii)  $\int \sin 3x \, dx$ .



### Example 4

If  $y = \sin 3x^2$ , find  $\frac{dy}{dx}$ .

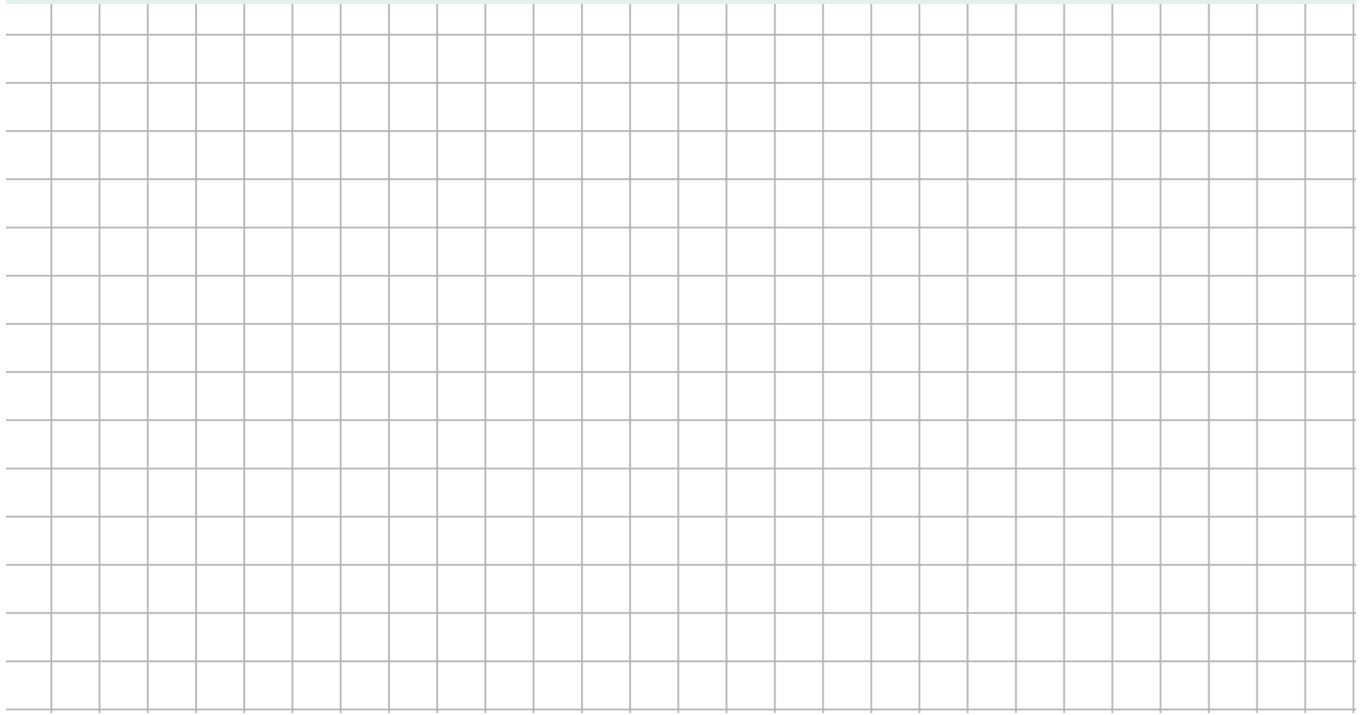


### Example 5

Let  $h(x) = x \ln x$ ,  $x \in R, x > 0$ .

(i) Find  $h'(x)$ .

(ii) Hence, find  $\int \ln x \, dx$ .



### Example 1

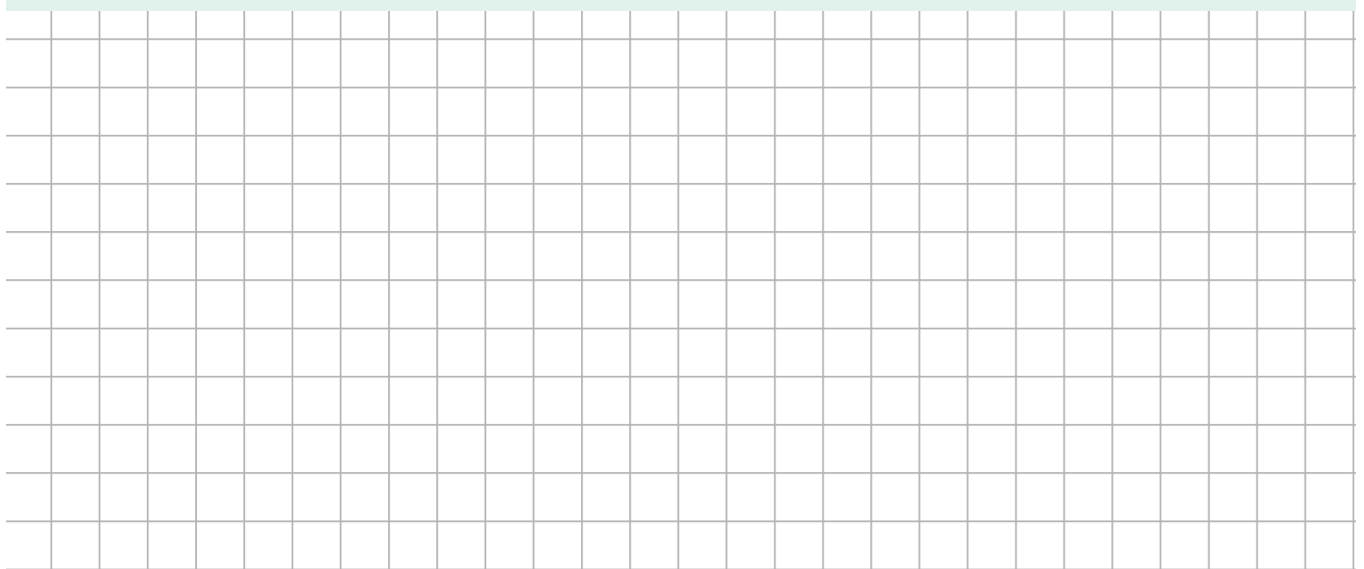
A body moves in a straight line.

At time  $t$  seconds, its acceleration is given by  $a = 6t + 1$ .

When  $t = 0$ , the velocity of the body is 2 m/sec and its displacement from a fixed point O is 1 metre.

(i) Find expressions for  $v$  and  $s$  in terms of  $t$ .

(ii) Find the velocity of the body after 4 seconds.



## Example 1

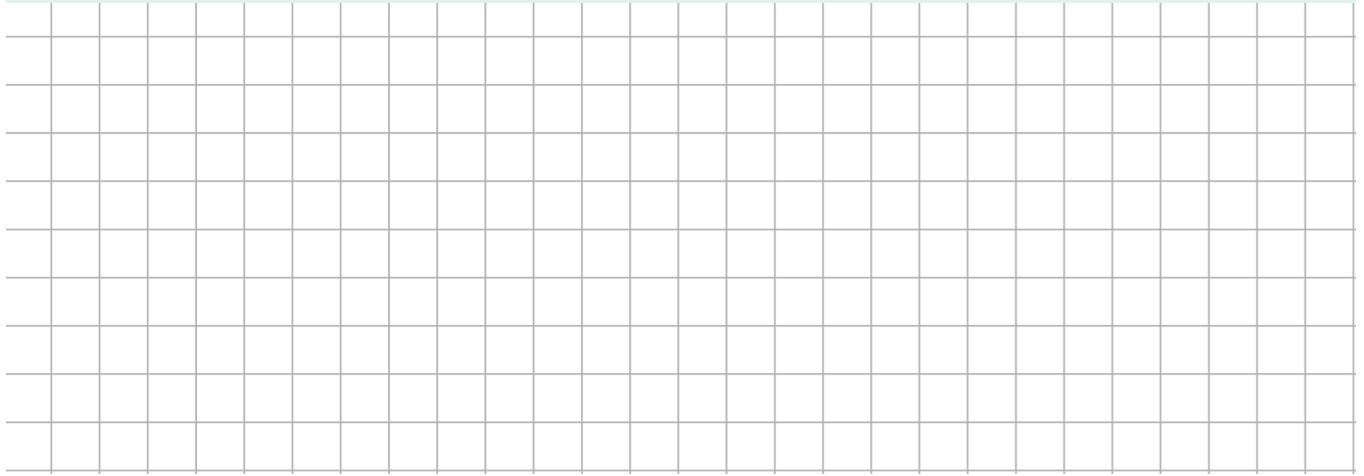
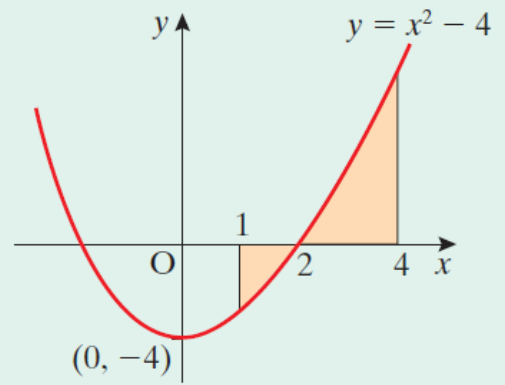
Evaluate (i)  $\int_0^2 3x^2 dx$       (ii)  $\int_2^4 (x^2 - x + 3) dx$       (iii)  $\int_4^9 \frac{1}{\sqrt{x}} dx$

## Example 2

Evaluate (i)  $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos 2x dx$       (ii)  $\int_2^5 4e^x dx$       (iii)  $\int_0^2 9^x dx$

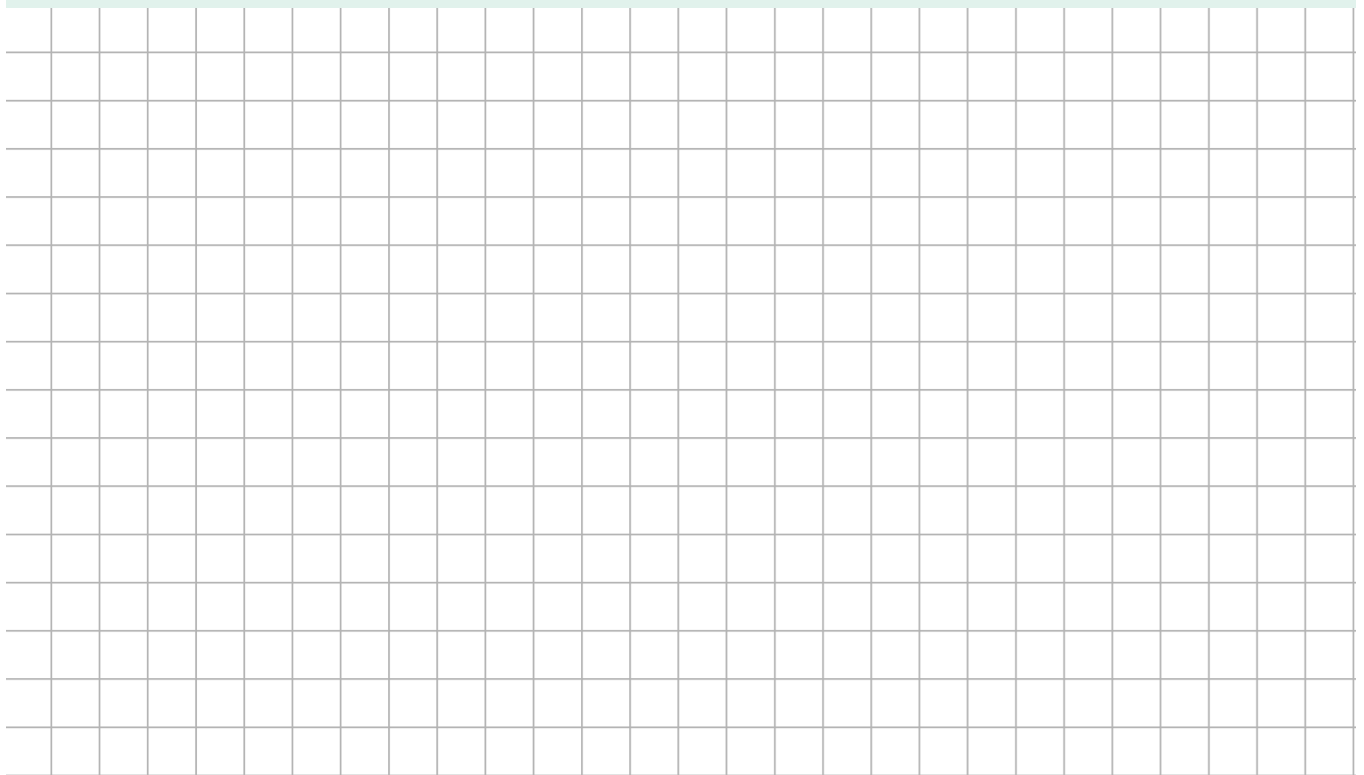
### Example 1

Find the area of the shaded region shown in the given diagram.



### Example 2

Find the area of the region bounded by the curve  $y = -x^2 + 5x - 4$  and the line  $y = x - 1$ .

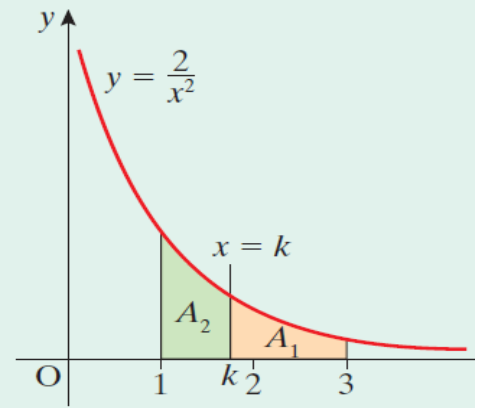


### Example 3

The diagram on the right shows a sketch of the function  $y = \frac{2}{x^2}$ .

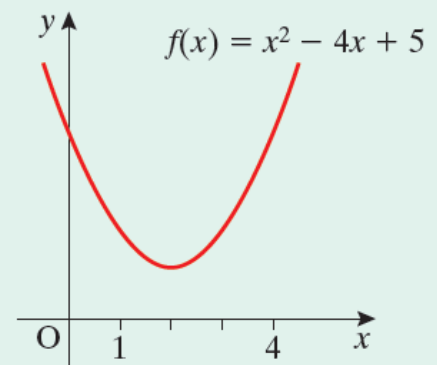
The shaded region represents the area bounded by the curve and the  $x$ -axis between the lines  $x = 3$  and  $x = 1$ .

If the line  $x = k$  divides this area into two equal portions, find the value of  $k$ .



### Example 1

The graph of the function,  $f(x) = x^2 - 4x + 5$  is shown. Find the average value of the function for  $1 \leq x \leq 4$ .





## Example 2

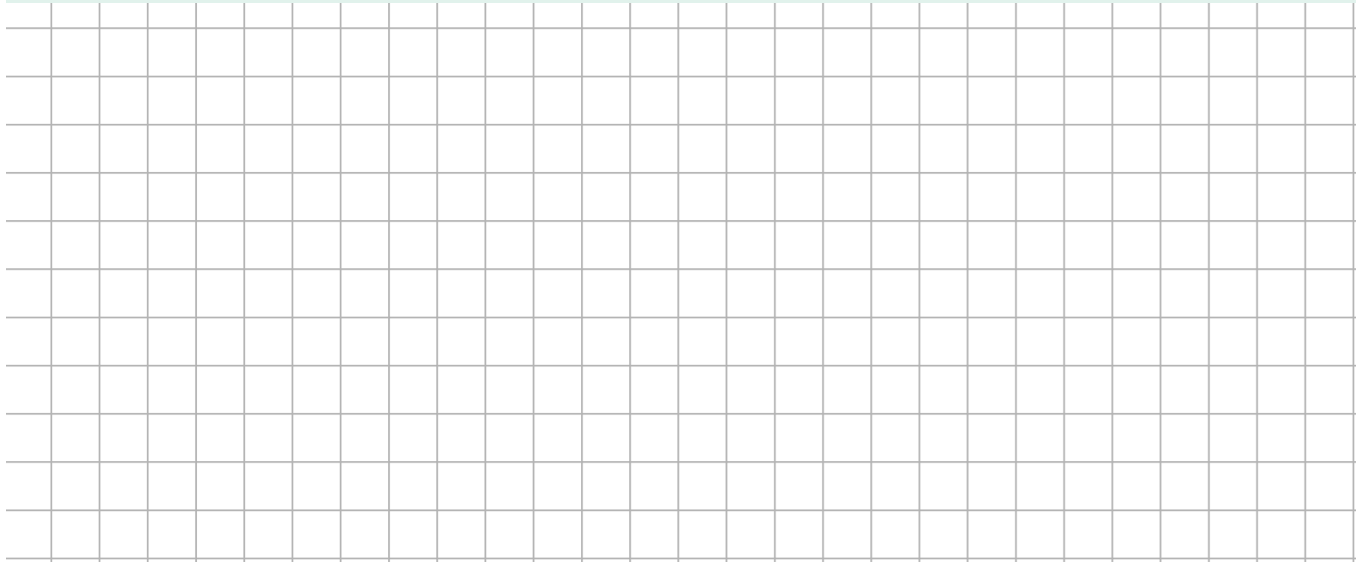
A body starts from rest and moves in a straight line.

After  $t$  seconds its velocity ( $v$ ) is given by  $v = 2t - 4, t \geq 0$ .

- (i) By completing the table on the right, find the average velocity over the first 3 seconds.

$t =$	0	1	2	3
$v =$				

- (ii) Use integration to test the accuracy of your answer.



## Example 3

The average value of the function  $f(x) = 2x + 3$  for  $1 \leq x \leq k$  is 11.

Find the value of  $k$ .

