Question 6 **(25 marks)**

(a) **(i)** Write down three distinct anti-derivatives of the function

$$g : x \mapsto x^3 - 3x^2 + 3, \qquad x \in \mathbb{R}.$$

1. $\frac{x^4}{4} - x^3 + 3x$
2. $\frac{x^4}{4} - x^3 + 3x + 1$
3. $\frac{x^4}{4} - x^3 + 3x + 2$

$\int gx.dx = \frac{x^4}{4} - \frac{\cancel{3}x^3}{\cancel{3}} + 3x + c$

(ii) Explain what is meant by the indefinite integral of a function *f*.

Indefinite integrals contain an undefined constant "C"

(b) **(i)** Let $h(x) = x \ln x$, for $x \in \mathbb{R}$, $x > 0$.
Find $h'(x)$.

Product Rule $f'(x) = u\frac{dv}{dx} + v\frac{du}{dx}$

let $u = x$

$\frac{du}{dx} = 1$

let $v = \ln x$

$\frac{dv}{dx} = \frac{1}{x}$

$h'(x) = x\left(\frac{1}{x}\right) + \ln x$

$h'(x) = 1 + \ln x$

(ii) Hence, find $\int \ln x \, dx$.

Since $h'(x) = 1 + \ln x$

$\Rightarrow \ln x = h'(x) - 1$

$\Rightarrow \int \ln x \, dx = \int (h'(x) - 1)\, dx = x.\ln x - x + c$