

Differentiation from first principles

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \quad \text{Definition}$$

Differentiate $f(x) = 2x^2 + 3x$ from 1st Principles

$$\begin{aligned} \textcircled{1} \quad f(x+h) &= 2(x+h)^2 + 3(x+h) \\ &= 2[x^2 + 2xh + h^2] + 3x + 3h \\ &= 2x^2 + 4xh + 2h^2 + 3x + 3h \end{aligned}$$

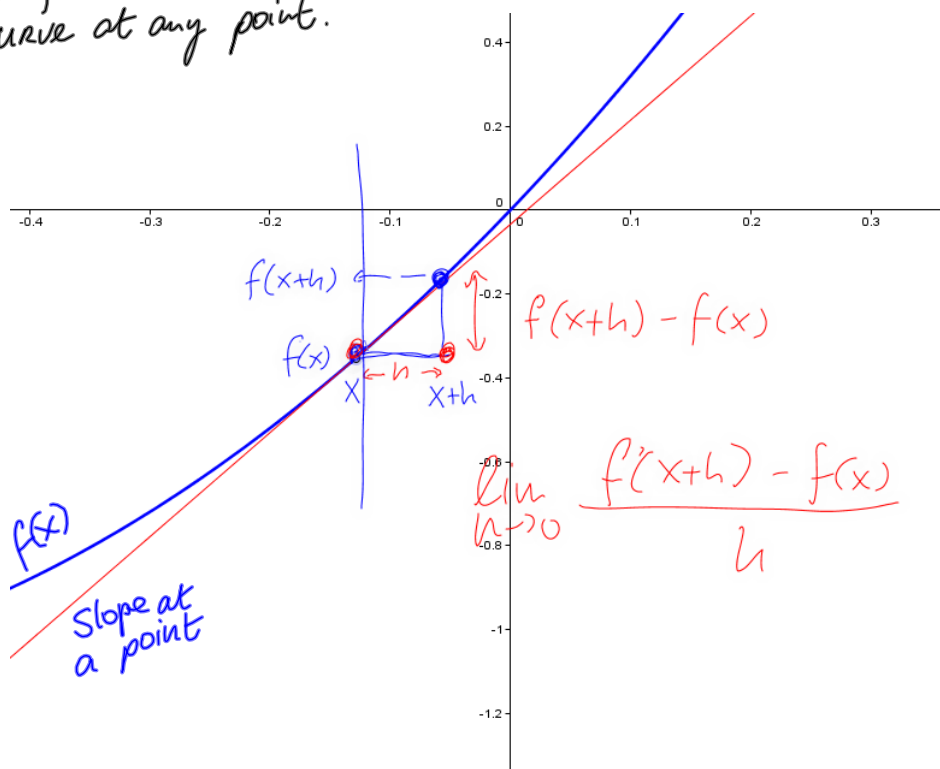
$$\begin{aligned} \textcircled{2} \quad \frac{f(x+h) - f(x)}{h} &= \frac{\cancel{2x^2} + 4x\cancel{h} + 2\cancel{h^2} + 3x + 3\cancel{h} - \cancel{2x^2} - \cancel{3x}}{h} \\ &= 4x + 2h + 3 \end{aligned}$$

$$\textcircled{3} \quad f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = 4x + 2(0) + 3 = 4x + 3$$

Differentiation is concerned with division of changing related terms.

This can be used to find the slope of a line.

Differentiation allows us to find the slope of a curve at any point.



HW Questions

Differentiate from first principles:

① $f(x) = 2x + 1$

② $f(x) = 4x^2 + 3x + 2$