Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination 2017

Mathematics

Paper 1

Higher Level

Friday 9 June   Afternoon 2:00 – 4:30

300 marks

Examination number

Centre stamp

For examiner	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

Running total	

Grade

Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	3 questions

Answer **all nine** questions.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so. You may ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

You will lose marks if all necessary work is not clearly shown.

You may lose marks if the appropriate units of measurement are not included, where relevant.

You may lose marks if your answers are not given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Section A Concepts and Skills 150 marks

Answer **all six** questions from this section.

Question 1 **(25 marks)**

(a) Write the function $f(x) = 2x^2 - 7x - 10$, where $x \in \mathbb{R}$, in the form $a(x+h)^2 + k$, where a, h, and $k \in \mathbb{Q}$.

(b) Hence, write the minimum point of f.

(c) **(i)** Explain why f must have two real roots.

(ii) Write the roots of $f(x) = 0$ in the form $p \pm \sqrt{q}$, where p and $q \in \mathbb{Q}$.

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Question 2 **(25 marks)**

$z = -\sqrt{3} + i$, where $i^2 = -1$.

(a) Use De Moivre's Theorem to write z^4 in the form $a + b\sqrt{c}\,i$, where a, b, and $c \in \mathbb{Z}$.

(b) The complex number w is such that $|w| = 3$ and w makes an angle of $30°$ with the positive sense of the real axis. If $t = zw$, write t in its simplest form.

Question 3 **(25 marks)**

(a) Differentiate $\frac{1}{3}x^2 - x + 3$ from first principles with respect to x.

(b) $f(x) = \ln(3x^2 + 2)$ and $g(x) = x + 5$, where $x \in \mathbb{R}$.

Find the value of the derivative of $f\left(g(x)\right)$ at $x = \frac{1}{4}$.

Give your answer correct to 3 decimal places.

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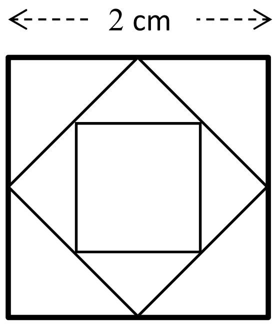
Question 4 **(25 marks)**

(a) The amount of a substance remaining in a solution reduces exponentially over time. An experiment measures the percentage of the substance remaining in the solution. The percentage is measured at the same time each day. The data collected over the first 4 days are given in the table below. Based on the data in the table, estimate which is the first day on which the percentage of the substance in the solution will be less than 0·01%.

Day	**1**	**2**	**3**	**4**
Percentage of substance (%)	95	42·75	19·2375	8·6569

(b) A square has sides of length 2 cm. The midpoints of the sides of this square are joined to form another square. This process is continued.
The first three squares in the process are shown below.
Find the sum of the perimeters of the squares if this process is continued indefinitely.
Give your answer in the form $a + b\sqrt{c}$ cm, where a, b, and $c \in \mathbb{N}$.



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Question 5 **(25 marks)**

The function f is such that $f(x) = 2x^3 + 5x^2 - 4x - 3$, where $x \in \mathbb{R}$.

(a) Show that $x = -3$ is a root of $f(x)$ **and** find the other two roots.

(b) Find the co-ordinates of the local maximum point **and** the local minimum point of the function f.

Local maximum point:

Local minimum point:

(c) $f(x) + a$, where a is a constant, has only one real root.
Find the range of possible values of a.

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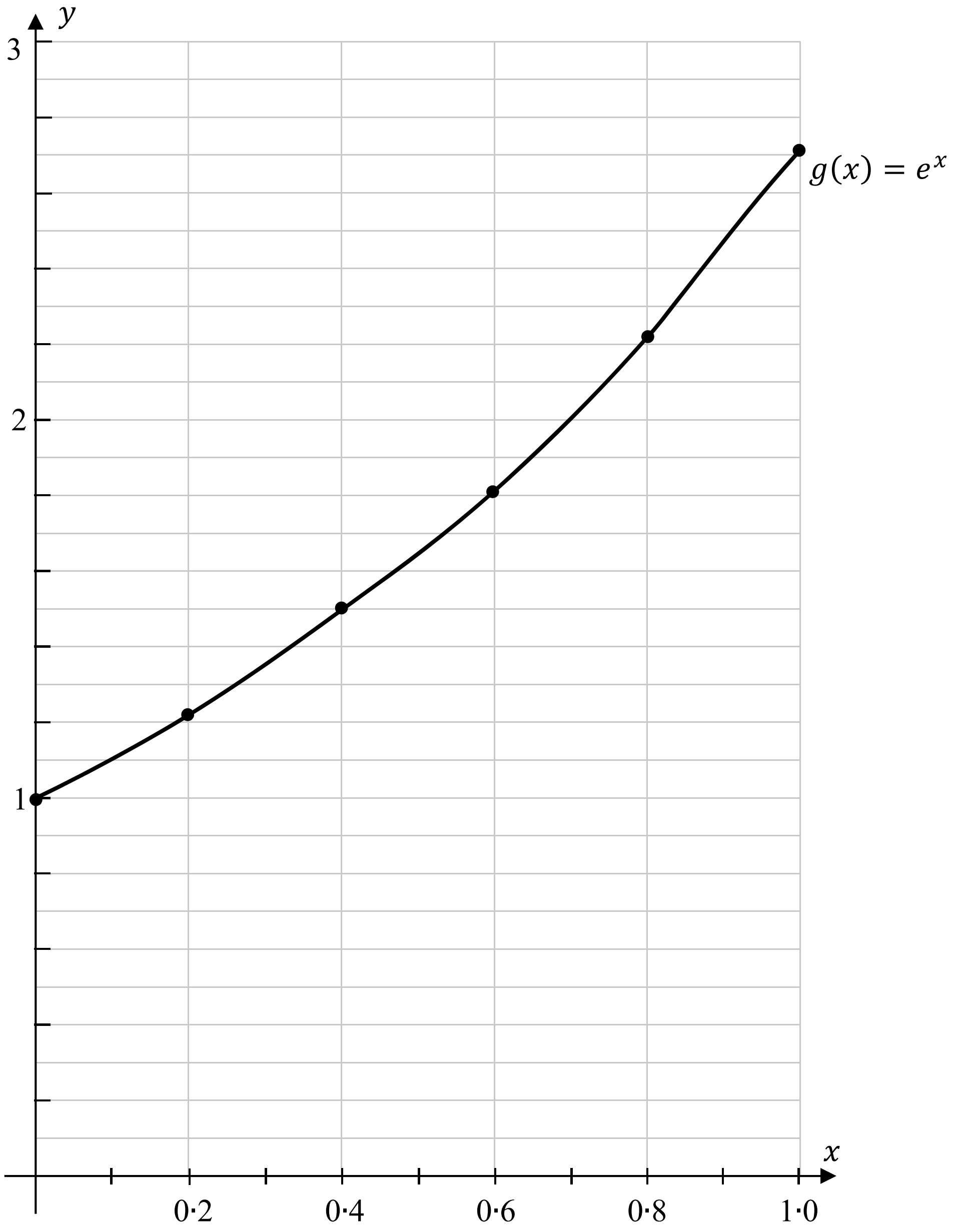
Question 6 **(25 marks)**

The graph of the function $g(x) = e^x$, $x \in \mathbb{R}$, $0 \leq x \leq 1$, is shown on the diagram below.

(a) On the same diagram, draw the graph of $h(x) = e^{-x}$, $x \in \mathbb{R}$, in the domain $0 \leq x \leq 1$.



(b) Find the area enclosed by $g(x) = e^x$, $h(x) = e^{-x}$, and the line $x = 0{\cdot}75$.
Give your answer correct to 4 decimal places.

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Section B   Contexts and Applications   150 marks

Answer **all three** questions from this section.

Question 7 **(55 marks)**

Sometimes it is possible to predict the future population in a city using a function.
The population in Sapphire City, over time, can be predicted using the following function:

$$p(t) = Se^{0\cdot 1t} \times 10^6.$$

The population in Avalon, over time, can be predicted using the following function:

$$q(t) = 3\cdot 9e^{kt} \times 10^6.$$

In the functions above, t is time, in years; $t = 0$ is the beginning of 2010; and both S and k are constants.

(a) The population in Sapphire City at the beginning of 2010 is 1 100 000 people.
Find the value of S.

(b) Find the predicted population in Sapphire City at the beginning of 2015.

(c) Find the predicted change in the population in Sapphire City during 2015.

(d) The predicted population in Avalon at the beginning of 2011 is 3 709 795 people. Write down and solve an equation in k to show that $k = -0{\cdot}05$, correct to 2 decimal places.

(e) Find the year during which the populations in both cities will be equal.

(f) Find the predicted average population in Avalon from the beginning of 2010 to the beginning of 2025.

This question is continued on the next page

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(g) Use the function $q(t) = 3{\cdot}9e^{-0{\cdot}05t} \times 10^6$ to find the predicted rate of change of the population in Avalon at the beginning of 2018.

Question 8 **(55 marks)**

(a) When a loan of €P is repaid in equal repayments of amount €A, at the end of each of t equal periods of time, where i is the periodic compound interest rate (expressed as a decimal), the formula below can be used to find the amount of each repayment.

$$A = P\frac{i(1+i)^t}{((1+i)^t - 1)}$$

Show how this formula is derived. You may use the formula for the sum of a finite geometric series.

This question is continued on the next page

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(b) Alex has a credit card debt of €5000. One method of clearing this debt is to make a fixed repayment at the end of each month. The amount of this repayment is 2·5% of the original debt.

(i) What is the fixed monthly repayment, €A, required to pay the debt of €5000?

(ii) The annual percentage rate (APR) charged on debt by the credit card company is 21·75%, fixed for the term of the debt. Find as a percentage, correct to 3 significant figures, the monthly interest rate that is equivalent to an APR of 21·75%.

(iii) Assume Alex pays the fixed monthly repayment, €A, each month and does not have any further transactions on that card. Complete the table below to show how the balance of the debt of €5000 is reducing each month for the first three months, assuming an APR of 21·75%, charged and compounded monthly.

Payment number	Fixed monthly payment, €A	€A		New balance of debt (€)
		Interest	Previous balance reduced by (€)	
0				5000
1			42·50	4957·50
2				
3				

(iv) Using the formula you derived on the previous page, or otherwise, find how long it would take to pay off a credit card debt of €5000, using the repayment method outlined at the beginning of **part (b)** above.
Give your answer in months, correct to the nearest month.

(v) Alex decides to borrow €5000 from the local Credit Union to pay off this credit card debt of €5000. The APR charge for the Credit Union loan is 8·5% fixed for the term of the loan. The loan is to be repaid in equal weekly repayments, at the end of each week, for 156 weeks. Find the amount of each weekly repayment.

(vi) How much will Alex save by paying off the credit card debt using the loan from the Credit Union instead of paying the fixed repayment from **part (b)(i)** each month to the credit card company?

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Question 9 **(40 marks)**

The depth of water, in metres, at a certain point in a harbour varies with the tide and can be modelled by a function of the form

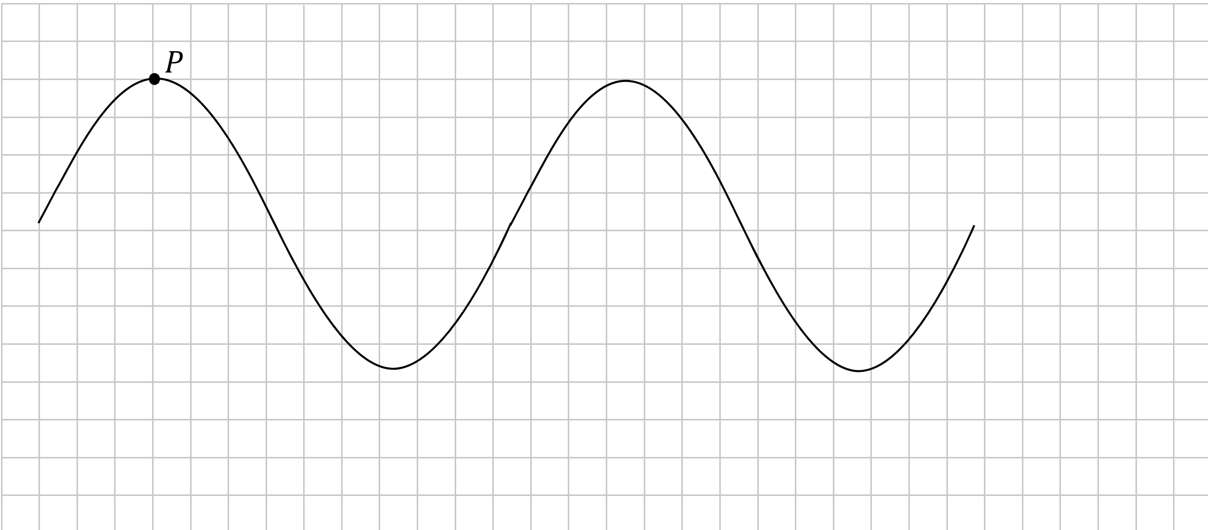
$$f(t) = a + b\cos ct$$

where t is the time in hours from the first high tide on a particular Saturday and a, b, and c are constants. (**Note**: ct is expressed in radians.)

On that Saturday, the following were noted:

- The depth of the water in the harbour at high tide was 5·5 m
- The depth of the water in the harbour at low tide was 1·7 m
- High tide occurred at 02:00 and again at 14:34.

(a) Use the information you are given to add, as accurately as you can, labelled and scaled axes to the diagram below to show the graph of f over a portion of that Saturday.
The point P should represent the depth of the water in the harbour at high tide on that Saturday morning.



(b) **(i)** Find the value of a and the value of b.

(ii) Show that $c = 0{\cdot}5$, correct to 1 decimal place.

(c) Use the equation $f(t) = a + b \cos ct$ to find the times on that Saturday **afternoon** when the depth of the water in the harbour was exactly 5·2 m.
Give each answer correct to the nearest minute.

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