

Question 3

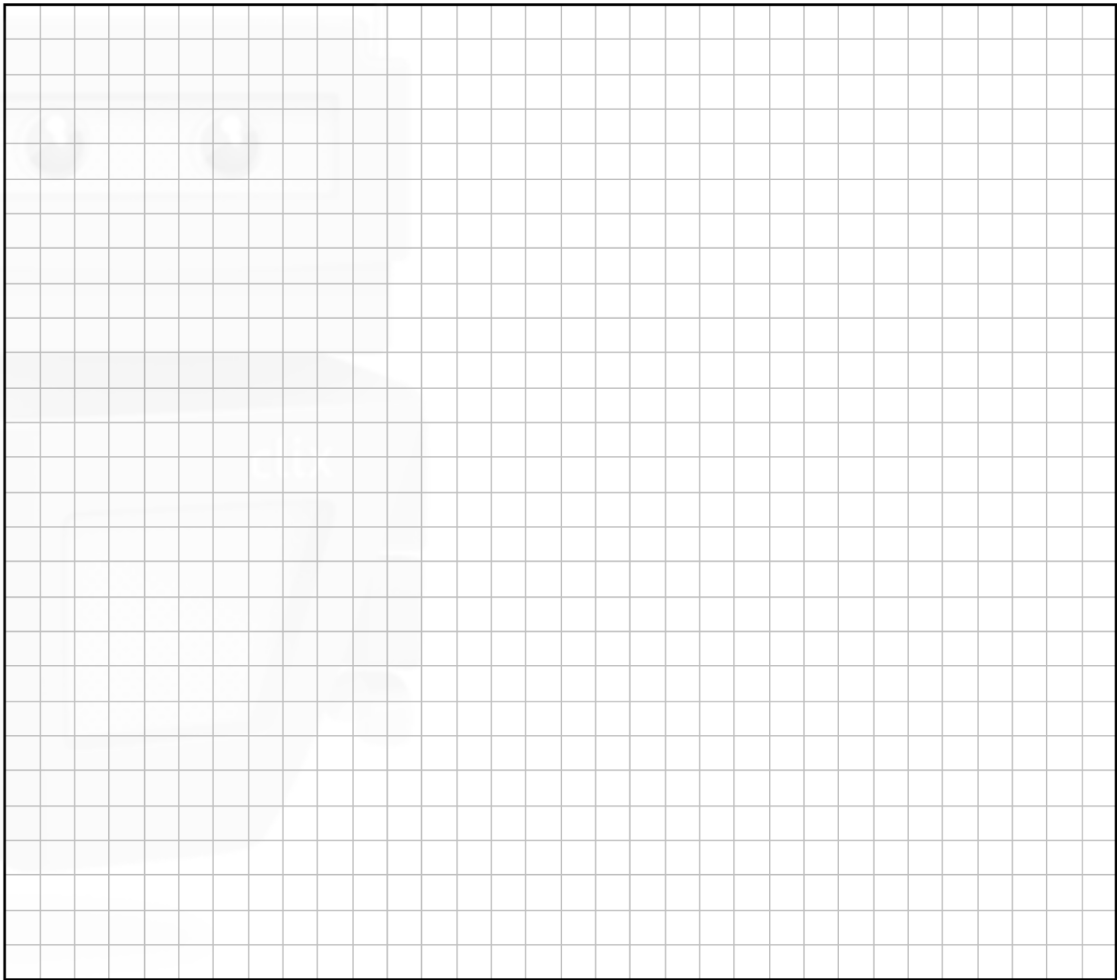
(25 marks)

- (a) Factorise fully: $3xy - 9x + 4y - 12$.

- (b) $g(x) = 3x \ln x - 9x + 4 \ln x - 12$.

Using your answer to **part (a)** or otherwise, solve $g(x) = 0$.

(c) Evaluate $g'(e)$ correct to 2 decimal places.



Marking Scheme



(a)	$(3x + 4)(y - 3)$	Scale 5B (0, 2, 5) <i>Mid Partial Credit:</i> - Any relevant factorisation
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(b)

$$3x \ln x - 9x + 4 \ln x - 12 =$$

$$3x(\ln x - 3) + 4(\ln x - 3) =$$

$$(3x + 4)(\ln x - 3)$$

$$3x + 4 = 0 \Rightarrow x = -\frac{4}{3} \text{ (not possible)}$$

$$\ln x - 3 = 0$$

$$\ln x = 3$$

$$x = e^3$$

Scale 10D (0, 4, 5, 8, 10)

Low Partial Credit:

- Any relevant factorisation of $g(x)$
- Trial and improvement with at least two values tested
- Substitutes $20 \leq x \leq 20.1$
- $y = \ln x$

Mid Partial Credit

- Expression fully factorised

High Partial Credit:

- $\ln x = 3$

Full Credit –1:

- Both solutions presented

Note: Accept $x = 20.1$ for $x = e^3$ in the last line of the solution

Note: If no reference is made to $3x + 4$ in the solution, then award high partial credit **at most**

(c)

$$g'(x) = 3x \left(\frac{1}{x} \right) + (3) \ln x - 9 + 4 \left(\frac{1}{x} \right)$$

$$g'(e) = 3(e) \left(\frac{1}{e} \right) + (3) \ln(e) - 9 + 4 \left(\frac{1}{e} \right)$$

$$g'(e) = 3 + 3 - 9 + \frac{4}{e} = -1.53$$

Scale 10D (0, 4, 5, 8, 10)

Low Partial Credit:

- Any relevant differentiation
- $g(e)$ evaluated correctly to at least 2 decimal places

Mid Partial Credit

- Expression fully differentiated
- Product rule not applied but finishes correctly

High Partial Credit:

- Derivative fully substituted

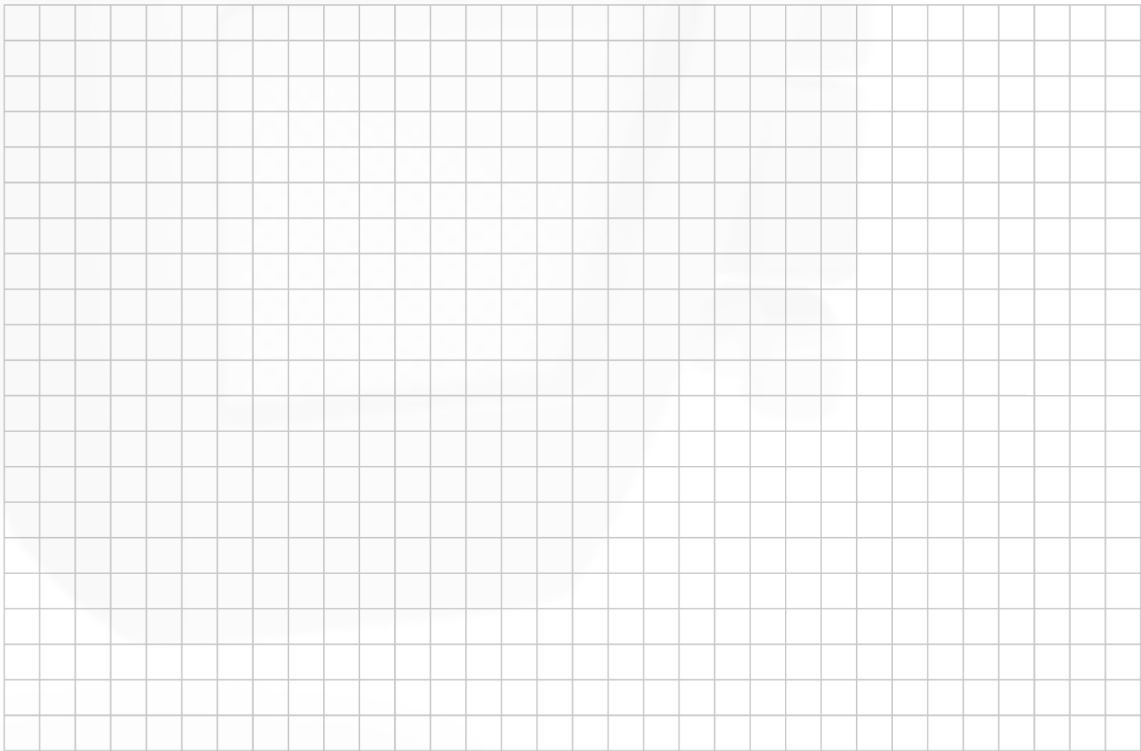
Question 3

(25 marks)

(a) Differentiate $\frac{1}{3}x^2 - x + 3$ from first principles with respect to x .



(b) $f(x) = \ln(3x^2 + 2)$ and $g(x) = x + 5$, where $x \in \mathbb{R}$.
Find the value of the derivative of $f(g(x))$ at $x = \frac{1}{4}$.
Give your answer correct to 3 decimal places.



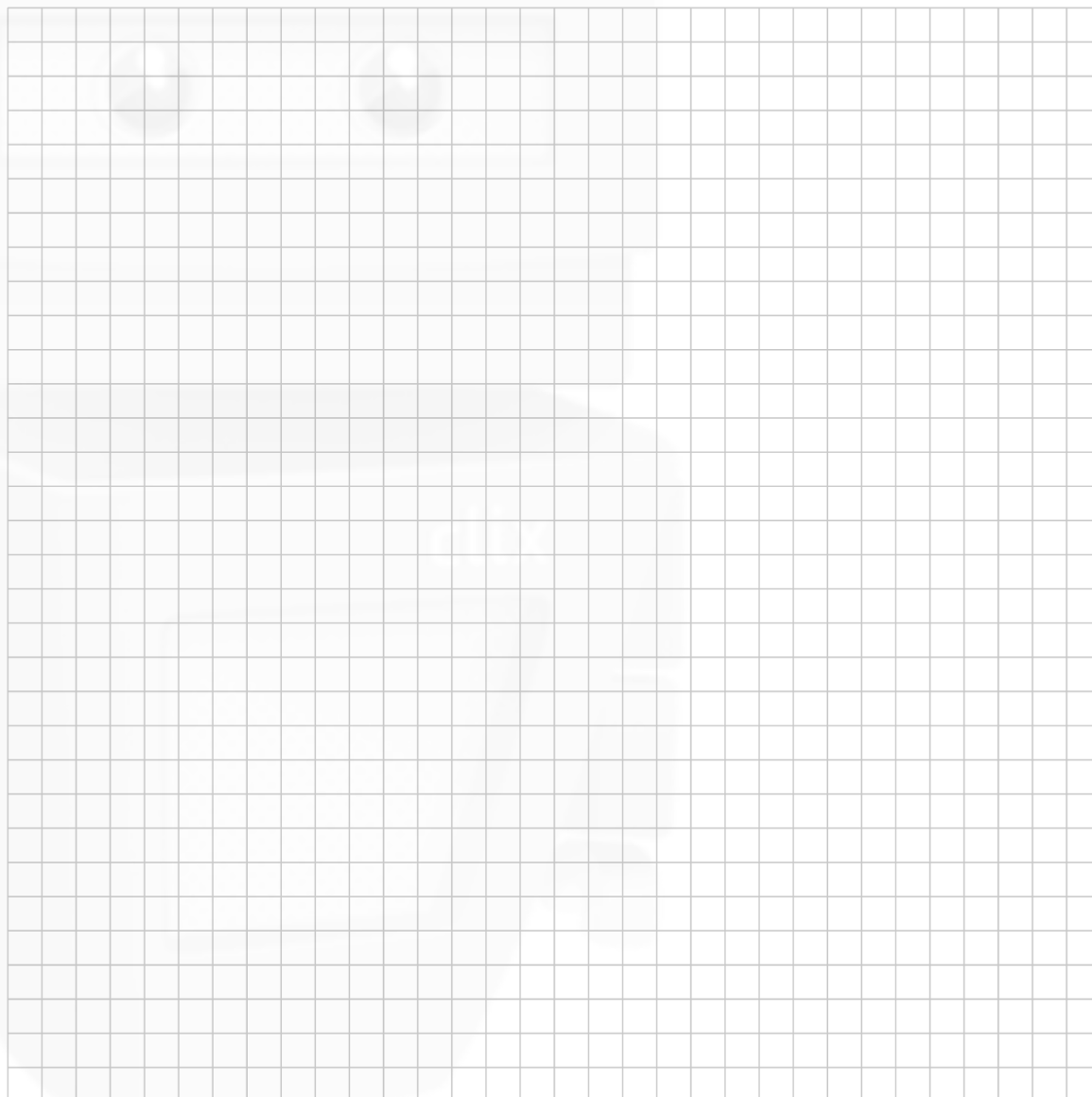
(a)	$f(x+h) = \frac{1}{3}(x+h)^2 - (x+h) + 3$ $f(x) = \frac{1}{3}x^2 - x + 3$ $f(x+h) - f(x) = \frac{2xh}{3} + \frac{h^2}{3} - h$ $\frac{f(x+h) - f(x)}{h} = \frac{2x}{3} + \frac{h}{3} - 1$ $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \frac{2x}{3} - 1$	<p>Scale 20D (0, 5, 14, 17, 20)</p> <p><i>Low Partial Credit</i></p> <ul style="list-style-type: none"> any $f(x+h)$ <p><i>Mid Partial Credit</i></p> <ul style="list-style-type: none"> $f(x+h) - f(x)$ with some correct work <p><i>High Partial Credit</i></p> <ul style="list-style-type: none"> $\frac{\frac{1}{3}(x+h)^2 - (x+h) + 3 - (\frac{x^2}{3} - x + 3)}{h}$ simplified <p>Notes:</p> <ul style="list-style-type: none"> omission of limit sign penalised once only answer not from 1st Principles merits 0 marks
(b)	$\frac{d(fg(x))}{dx} =$ $\frac{1}{(3(x+5)^2 + 2)} (6(x+5))$ $\frac{d(fg(\frac{1}{4}))}{dx} = \frac{6(\frac{21}{4})}{3(\frac{21}{4})^2 + 2} = \frac{504}{1355}$ $= 0.372$ <p style="text-align: center;">OR</p> $f(x) = \ln(3x^2 + 2)$ $g(x) = (x+5)$ $f[g(x)] = \ln[3(x+5)^2 + 2]$ $= \ln(3x^2 + 30x + 77)$ $f'(x) = \frac{6x + 30}{3x^2 + 30x + 77}$ $x = \frac{1}{4}: f'(x) = \frac{31.5}{84.6875} = 0.3719$ $= 0.372$	<p>Scale 5C (0, 3, 4, 5)</p> <p><i>Low Partial Credit:</i></p> <ul style="list-style-type: none"> Any correct differentiation $fg(x)$ formulated <p><i>High Partial Credit:</i></p> <ul style="list-style-type: none"> $\frac{d(fg(x))}{dx}$ found <p>Note:</p> <p>Work with $f(x) \times g(x)$ merits low partial credit at most</p>

<p>(a)</p>	$r = \frac{42.75}{95} = \frac{9}{20} \quad T_n = ar^{n-1} < 0.01$ $95 \left(\frac{9}{20} \right)^{n-1} < 0.01$ $\left(\frac{9}{20} \right)^{n-1} < \frac{0.01}{95}$ $(n-1) \log \left(\frac{9}{20} \right) < \log \left(\frac{0.01}{95} \right)$ $(n-1) > \frac{\log \left(\frac{0.01}{95} \right)}{\log \left(\frac{9}{20} \right)}$ <p>(since $\log \left(\frac{9}{20} \right)$ is negative)</p> $n-1 > 11.47$ $n > 12.47$ <p>12th day</p>	<p>Scale 15D (0, 5, 8, 12, 15)</p> <p><i>Low Partial Credit:</i></p> <ul style="list-style-type: none"> • r found • T_n of a GP with some substitution <p><i>Mid Partial Credit:</i></p> <ul style="list-style-type: none"> • Inequality in n written <p><i>High Partial Credit:</i></p> <ul style="list-style-type: none"> • Inequality in n simplified (log handled) <p><i>Full Credit:</i></p> <ul style="list-style-type: none"> • Accept $n = 12.47$
<p>(b)</p>	$4(2) + 4\sqrt{2} + 4 + \dots$ $a = 8 \quad r = \frac{1}{\sqrt{2}}$ $S_{\infty} = \frac{a}{1-r}$ $S_{\infty} = \frac{8}{1 - \frac{1}{\sqrt{2}}}$ $S_{\infty} = \frac{8}{1 - \frac{1}{\sqrt{2}}} \cdot \frac{1 + \frac{1}{\sqrt{2}}}{1 + \frac{1}{\sqrt{2}}}$ $S_{\infty} = \frac{8 \left(1 + \frac{1}{\sqrt{2}} \right)}{\frac{1}{2}}$ $S_{\infty} = 16 + 8\sqrt{2}$	<p>Scale 10C (0, 5, 8, 10)</p> <p><i>Low Partial Credit:</i></p> <ul style="list-style-type: none"> • length of one side of new square <p><i>High Partial Credit:</i></p> <ul style="list-style-type: none"> • S_{∞} fully substituted • Correct work with one side only

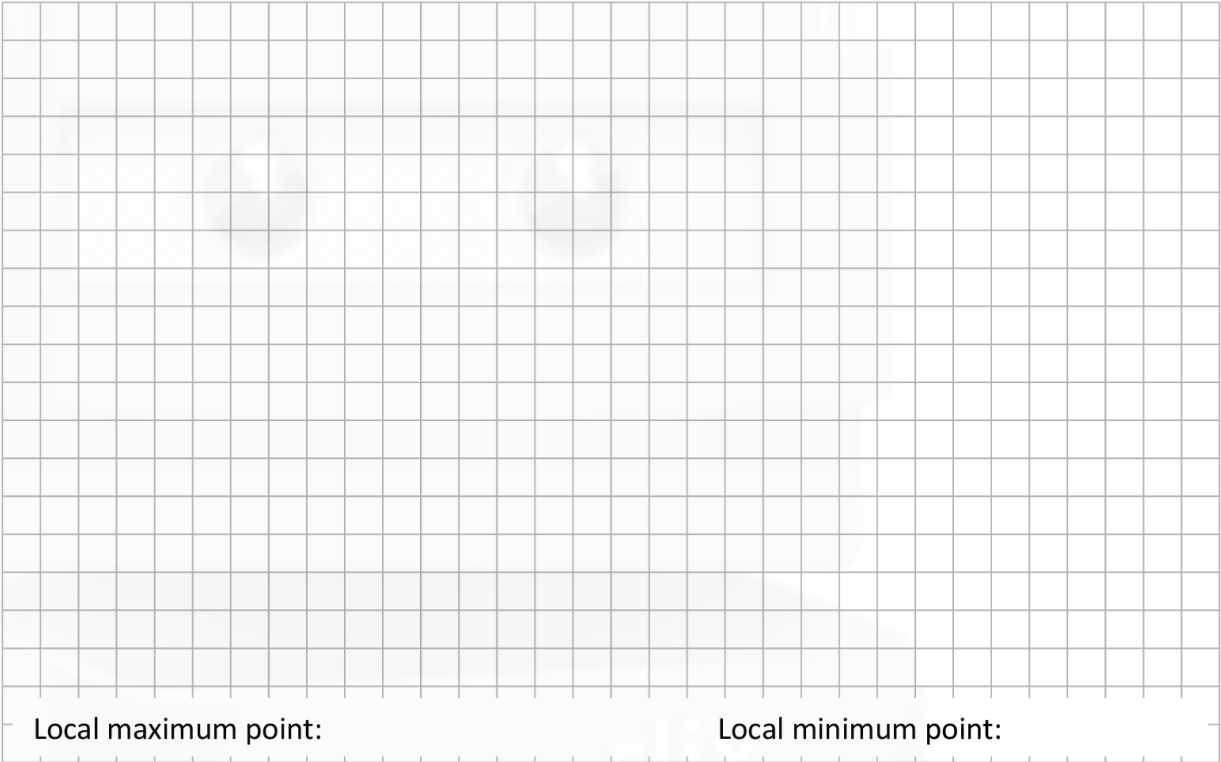
Question 5**(25 marks)**

The function f is such that $f(x) = 2x^3 + 5x^2 - 4x - 3$, where $x \in \mathbb{R}$.

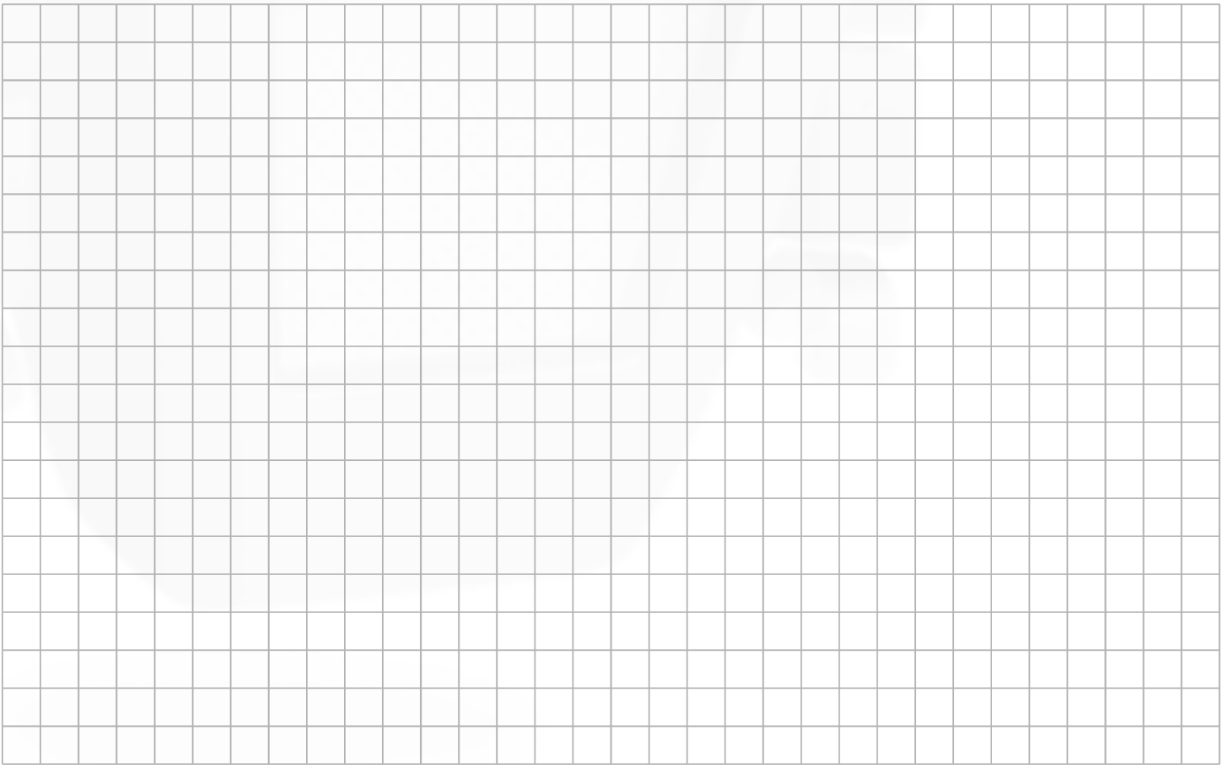
- (a)** Show that $x = -3$ is a root of $f(x)$ **and** find the other two roots.



- (b) Find the co-ordinates of the local maximum point **and** the local minimum point of the function f .



- (c) $f(x) + a$, where a is a constant, has only one real root.
Find the range of possible values of a .

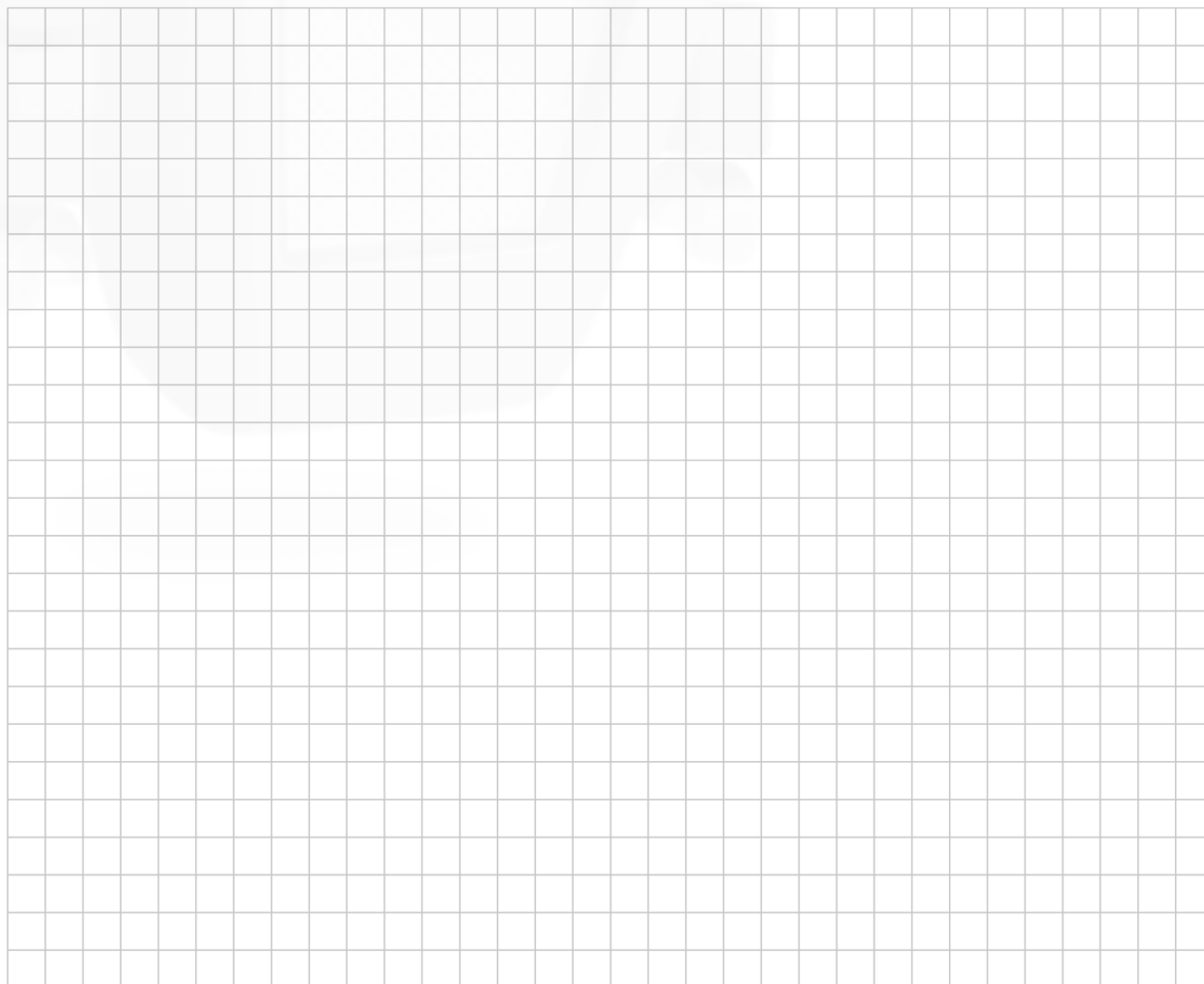


(a)	$f(x) = 2x^3 + 5x^2 - 4x - 3$ $f(-3) = 2(-3)^3 + 5(-3)^2 - 4(-3) - 3$ $= -54 + 45 + 12 - 3$ $f(-3) = 0$ $\Rightarrow (x + 3) \text{ is a factor}$ $ \begin{array}{r} 2x^2 - x - 1 \\ x+3 \overline{) 2x^3 + 5x^2 - 4x - 3} \\ \underline{2x^3 + 6x^2} \\ -x^2 - 4x \\ \underline{-x^2 - 3x} \\ -x - 3 \\ \underline{-x - 3} \\ 0 \end{array} $ $f(x) = (x + 3)(2x^2 - x - 1)$ $f(x) = (x + 3)(2x + 1)(x - 1)$ $x = -3 \quad x = -\frac{1}{2} \quad x = 1$	<p>Scale 15C (0, 5, 10, 15)</p> <p><i>Low Partial Credit:</i></p> <ul style="list-style-type: none"> Shows $f(-3) = 0$ <p><i>High Partial Credit:</i></p> <ul style="list-style-type: none"> quadratic factor of $f(x)$ found <p>Note: No remainder in division may be stated as reason for $x = -3$ as root</p>
(b)	$y = 2x^3 + 5x^2 - 4x - 3$ $\frac{dy}{dx} = 6x^2 + 10x - 4 = 0$ $3x^2 + 5x - 2 = 0$ $(x + 2)(3x - 1) = 0$ $3x - 1 = 0 \quad x + 2 = 0$ $x = \frac{1}{3} \quad x = -2$ $f\left(\frac{1}{3}\right) = \frac{-100}{27} \quad f(-2) = 9$ $Max = (-2, 9) \quad Min = \left(\frac{1}{3}, \frac{-100}{27}\right)$	<p>Scale 5C (0, 3, 4, 5)</p> <p><i>Low Partial Credit:</i></p> <ul style="list-style-type: none"> $\frac{dy}{dx}$ found (Some correct differentiation) <p><i>High Partial Credit</i></p> <ul style="list-style-type: none"> roots and one y value found <p>Note: One of Max/Min must be identified for full credit</p>
(c)	$a > \frac{100}{27} \quad \text{or} \quad a < -9$	<p>Scale 5B (0, 3, 5)</p> <p><i>Partial Credit:</i></p> <ul style="list-style-type: none"> one value identified no range identified (from 2 values)

- (a) Differentiate the function $(2x + 4)^2$ from first principles, with respect to x .

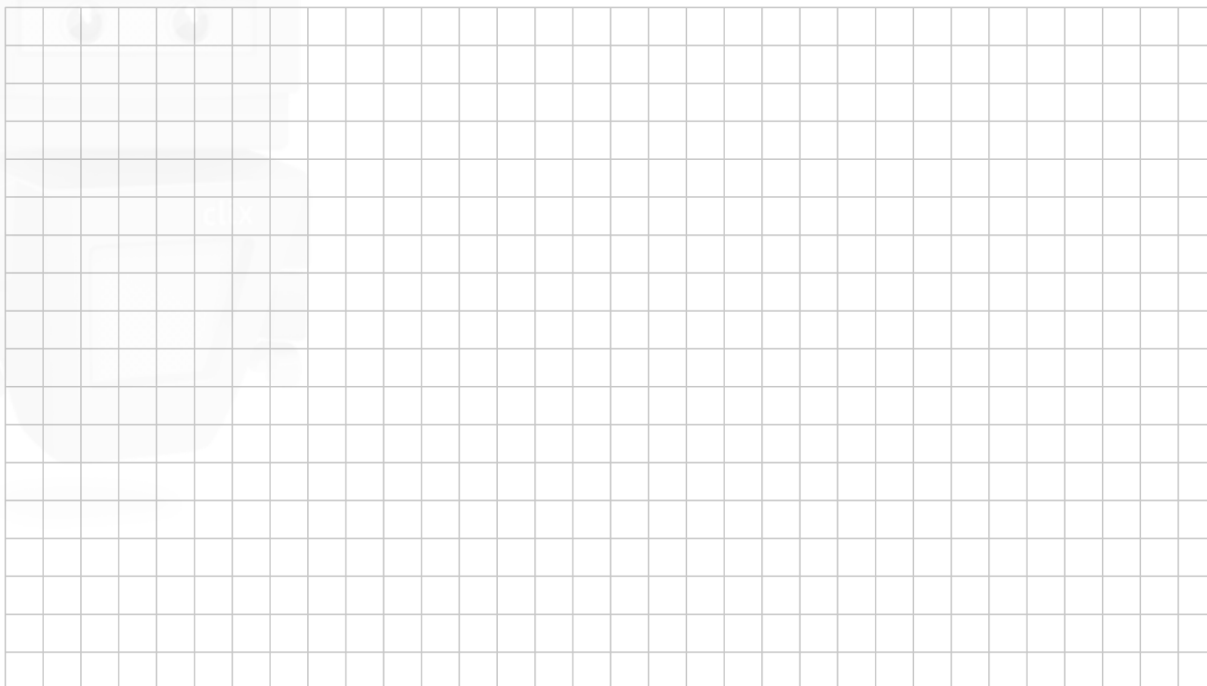


- (b) (i) If $y = x \sin\left(\frac{1}{x}\right)$, find $\frac{dy}{dx}$.



- (ii) Find the slope of the tangent to the curve $y = x \sin \left(\frac{1}{x} \right)$, when $x = \frac{4}{\pi}$.

Give your answer correct to two decimal places.



Marking Scheme

Q6	Model Solution – 25 Marks	Marking Notes
(a)	$f(x+h) - f(x) = (2x+2h+4)^2 - (2x+4)^2$ $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} =$ $\lim_{h \rightarrow 0} \frac{(2x+2h+4)^2 - (2x+4)^2}{h}$ $= \lim_{h \rightarrow 0} \left(\frac{[(4x^2 + 8hx + 4h^2 + 16x + 16h + 16) - (4x^2 + 16x + 16)]}{h} \right)$ $= \lim_{h \rightarrow 0} \frac{8hx + 4h^2 + 16h}{h}$ $= 8x + 16$ <p style="text-align: center;">or</p> $f(x) = (2x+4)^2 = 4x^2 + 16x + 16$ $f(x+h) = 4(x+h)^2 + 16(x+h) + 16$ $= 4x^2 + 8hx + 4h^2 + 16x + 16h + 16$ $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $\lim_{h \rightarrow 0} \frac{8hx + 4h^2 + 16h}{h}$ $= 8x + 16$	<p>Scale 10D (0, 2, 5, 8, 10)</p> <p><i>Low Partial Credit</i></p> <ul style="list-style-type: none"> any $f(x+h)$ <p><i>Mid Partial Credit</i></p> <ul style="list-style-type: none"> limit of $\frac{f(x+h) - f(x)}{h}$ <p><i>High Partial Credit</i></p> <ul style="list-style-type: none"> limit of $\frac{(2x+2h+4)^2 - (2x+4)^2}{h}$ <p>Notes:</p> <ul style="list-style-type: none"> omission of limit sign penalised once only answer not from 1st Principles merits 0 marks

- (b) Differentiate $x - \sqrt{x+6}$ with respect to x .

$$f(x) = x - \sqrt{x+6} = x - (x+6)^{\frac{1}{2}}$$

$$f'(x) = 1 - \frac{1}{2}(x+6)^{-\frac{1}{2}} = 1 - \frac{1}{2\sqrt{x+6}}$$

- (c) Find the co-ordinates of the turning point of the function $y = x - \sqrt{x+6}$, $x \geq -6$.

$$f'(x) = 0 \Rightarrow 1 - \frac{1}{2\sqrt{x+6}} = 0$$

$$\Rightarrow 2\sqrt{x+6} = 1$$

$$\Rightarrow x+6 = \frac{1}{4}$$

$$\Rightarrow x = -5\frac{3}{4}$$

$$f(-5\frac{3}{4}) = -5\frac{3}{4} - \sqrt{\frac{1}{4}} = -6\frac{1}{4}$$

$$(-5\frac{3}{4}, -6\frac{1}{4})$$

