Question 3 **(25 marks)**

(a) Factorise fully: $3xy - 9x + 4y - 12$.

(b) $g(x) = 3x\ln x - 9x + 4\ln x - 12$.
Using your answer to **part (a)** or otherwise, solve $g(x) = 0$.

(c) Evaluate $g'(e)$ correct to 2 decimal places.

Marking Scheme

(a)	$(3x+4)(y-3)$	**Scale 5B (0, 2, 5)** *Mid Partial Credit:* - Any relevant factorisation

(b)	$3xlnx - 9x + 4lnx - 12 =$ $3x(lnx - 3) + 4(lnx - 3) =$ $(3x + 4)(lnx - 3)$ $3x + 4 = 0 \Rightarrow x = -\frac{4}{3}$ (not possible) $lnx - 3 = 0$ $lnx = 3$ $x = e^3$	**Scale 10D (0, 4, 5, 8, 10)** *Low Partial Credit:* - Any relevant factorisation of $g(x)$ - Trial and improvement with at least two values tested - Substitutes $20 \le x \le 20{\cdot}1$ - $y = lnx$ *Mid Partial Credit* - Expression fully factorised *High Partial Credit:* - $lnx = 3$ *Full Credit –1:* - Both solutions presented <u>*Note*</u>: Accept $x = 20{\cdot}1$ for $x = e^3$ in the last line of the solution <u>*Note*</u>: If no reference is made to $3x + 4$ in the solution, then award high partial credit **at most**
(c)	$g'(x) = 3x\left(\frac{1}{x}\right) + (3)lnx - 9 + 4\left(\frac{1}{x}\right)$ $g'(e) = 3(e)\left(\frac{1}{e}\right) + (3)\,ln(e) - 9 + 4\left(\frac{1}{e}\right)$ $g'(e) = 3 + 3 - 9 + \frac{4}{e} = -1{\cdot}53$	**Scale 10D (0, 4, 5, 8, 10)** *Low Partial Credit:* - Any relevant differentiation - $g(e)$ evaluated correctly to at least 2 decimal places *Mid Partial Credit* - Expression fully differentiated - Product rule not applied but finishes correctly *High Partial Credit:* - Derivative fully substituted

Question 3 **(25 marks)**

(a) Differentiate $\frac{1}{3}x^2 - x + 3$ from first principles with respect to x.

(b) $f(x) = \ln(3x^2 + 2)$ and $g(x) = x + 5$, where $x \in \mathbb{R}$.

Find the value of the derivative of $f\big(g(x)\big)$ at $x = \frac{1}{4}$.

Give your answer correct to 3 decimal places.

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(a)	$f(x+h)=\frac{1}{3}(x+h)^2-(x+h)+3$ $f(x)=\frac{1}{3}x^2-x+3$ $f(x+h)-f(x)=\frac{2xh}{3}+\frac{h^2}{3}-h$ $\frac{f(x+h)-f(x)}{h}=\frac{2x}{3}+\frac{h}{3}-1$ $\lim\limits_{h\to 0}\frac{f(x+h)-f(x)}{h}=\frac{2x}{3}-1$	**Scale 20D (0, 5, 14, 17, 20)** *Low Partial Credit* • any $f(x+h)$ *Mid Partial Credit* • $f(x+h)-f(x)$ *with some correct work* *High Partial Credit* • $\frac{\frac{1}{3}(x+h)^2-(x+h)+3-\left(\frac{x^2}{3}-x+3\right)}{h}$ simplified **Notes***:* • omission of limit sign penalised once only • answer not from 1st Principles merits 0 marks
(b)	$\frac{d(fg(x))}{dx}=$ $\frac{1}{(3(x+5)^2+2)}(6(x+5))$ $\frac{d\left(fg\left(\frac{1}{4}\right)\right)}{dx}=\frac{6(\frac{21}{4})}{3(\frac{21}{4})^2+2}=\frac{504}{1355}$ $=0{\cdot}372$ **OR** $f(x)=\ln(3x^2+2)$ $g(x)=(x+5)$ $f[g(x)]=\ln[3(x+5)^2+2]$ $=\ln(3x^2+30x+77)$ $f'(x)=\frac{6x+30}{3x^2+30x+77}$ $x=\frac{1}{4}$: $f'(x)=\frac{31{\cdot}5}{84{\cdot}6875}=0{\cdot}3719$ $=0{\cdot}372$	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* • Any correct differentiation • $fg(x)$ formulated *High Partial Credit:* • $\frac{d(fg(x))}{dx}$ found **Note***:* Work with $f(x)\times g(x)$ merits low partial credit at most

(a)	$r = \frac{42\cdot75}{95} = \frac{9}{20}$ $\quad T_n = ar^{n-1} < 0\cdot01$ $95\left(\frac{9}{20}\right)^{n-1} < 0\cdot01$ $\left(\frac{9}{20}\right)^{n-1} < \frac{0\cdot01}{95}$ $(n-1)\log\left(\frac{9}{20}\right) < \log\left(\frac{0.01}{95}\right)$ $(n-1) > \frac{\log\left(\frac{0.01}{95}\right)}{\log\left(\frac{9}{20}\right)}$ (since $\log\left(\frac{9}{20}\right)$ is negative) $n-1 > 11\cdot47$ $n > 12\cdot47$ 12th day	**Scale 15D (0, 5, 8, 12, 15)** *Low Partial Credit:* • r found • T_n of a GP with some substitution *Mid Partial Credit:* • Inequality in n written *High Partial Credit:* • Inequality in n simplified (log handled) *Full Credit:* • Accept $n = 12\cdot47$
(b)	$4(2) + 4\sqrt{2} + 4 + \cdots\cdots\cdots$ $a = 8 \quad r = \frac{1}{\sqrt{2}}$ $S_\infty = \frac{a}{1-r}$ $S_\infty = \frac{8}{1-\frac{1}{\sqrt{2}}}$ $S_\infty = \frac{8}{1-\frac{1}{\sqrt{2}}} \cdot \frac{1+\frac{1}{\sqrt{2}}}{1+\frac{1}{\sqrt{2}}}$ $S_\infty = \frac{8\left(1+\frac{1}{\sqrt{2}}\right)}{\frac{1}{2}}$ $S_\infty = 16 + 8\sqrt{2}$	**Scale 10C (0, 5, 8, 10)** *Low Partial Credit:* • length of one side of new square *High Partial Credit:* • S_∞ fully substituted • Correct work with one side only

Question 5 **(25 marks)**

The function f is such that $f(x) = 2x^3 + 5x^2 - 4x - 3$, where $x \in \mathbb{R}$.

(a) Show that $x = -3$ is a root of $f(x)$ **and** find the other two roots.

(b) Find the co-ordinates of the local maximum point **and** the local minimum point of the function f.

Local maximum point: Local minimum point:

(c) $f(x) + a$, where a is a constant, has only one real root.
Find the range of possible values of a.

(a)	$f(x) = 2x^3 + 5x^2 - 4x - 3$ $f(-3) = 2(-3)^3 + 5(-3)^2 - 4(-3) - 3$ $= -54 + 45 + 12 - 3$ $f(-3) = 0$ $\Rightarrow (x+3)$ is a factor $\begin{array}{r} 2x^2 - x - 1 \\ x+3\overline{)2x^3 + 5x^2 - 4x - 3} \\ \underline{2x^3 + 6x^2} \\ -x^2 - 4x \\ \underline{-x^2 - 3x} \\ -x - 3 \\ \underline{-x - 3} \end{array}$ $f(x) = (x+3)(2x^2 - x - 1)$ $f(x) = (x+3)(2x+1)(x-1)$ $x = -3 \quad x = -\frac{1}{2} \quad x = 1$	**Scale 15C (0, 5, 10, 15)** *Low Partial Credit:* • Shows $f(-3) = 0$ *High Partial Credit:* • quadratic factor of $f(x)$ found **Note:** No remainder in division may be stated as reason for $x = -3$ as root
(b)	$y = 2x^3 + 5x^2 - 4x - 3$ $\frac{dy}{dx} = 6x^2 + 10x - 4 = 0$ $3x^2 + 5x - 2 = 0$ $(x+2)(3x-1) = 0$ $3x - 1 = 0 \quad x + 2 = 0$ $x = \frac{1}{3} \quad x = -2$ $f\left(\frac{1}{3}\right) = \frac{-100}{27} \quad f(-2) = 9$ $Max = (-2, 9) \quad Min = \left(\frac{1}{3}, \frac{-100}{27}\right)$	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* • $\frac{dy}{dx}$ found (Some correct differentiation) *High Partial Credit* • roots and one y value found **Note:** One of Max/Min must be identified for full credit
(c)	$a > \frac{100}{27}$ **or** $a < -9$	**Scale 5B (0, 3, 5)** *Partial Credit:* • one value identified • no range identified (from 2 values)

(a) Differentiate the function $(2x+4)^2$ from first principles, with respect to x.

(b) **(i)** If $y = x\sin\left(\frac{1}{x}\right)$, find $\frac{dy}{dx}$.

(ii) Find the slope of the tangent to the curve $y = x\sin\left(\frac{1}{x}\right)$, when $x = \frac{4}{\pi}$.
Give your answer correct to two decimal places.

Marking Scheme

Q6	Model Solution – 25 Marks	Marking Notes
(a)	$f(x+h) - f(x) = (2x+2h+4)^2 - (2x+4)^2$ $\lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h} =$ $\lim\limits_{h\to 0} \dfrac{(2x+2h+4)^2 - (2x+4)^2}{h}$ $= \lim\limits_{h\to 0}\left(\dfrac{\begin{bmatrix}(4x^2+8hx+4h^2+16x+16h+16)\\ -(4x^2+16x+16)\end{bmatrix}}{h}\right)$ $= \lim\limits_{h\to 0} \dfrac{8hx+4h^2+16h}{h}$ $= 8x+16$ **or** $f(x) = (2x+4)^2 = 4x^2+16x+16$ $f(x+h) = 4(x+h)^2 + 16(x+h) + 16$ $= 4x^2+8hx+4h^2+16x+16h+16$ $\lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$ $\lim\limits_{h\to 0} \dfrac{8hx+4h^2+16h}{h}$ $= 8x+16$	Scale 10D (0, 2, 5, 8, 10) *Low Partial Credit* • any $f(x+h)$ *Mid Partial Credit* • limit of $\dfrac{f(x+h)-f(x)}{h}$ *High Partial Credit* • limit of $\dfrac{(2x+2h+4)^2-(2x+4)^2}{h}$ **Notes:** - *omission of limit sign penalised once only* - *answer not from 1st Principles merits 0 marks*

(b) (i)+ (ii)	$y = x.\sin\frac{1}{x}$ $\frac{dy}{dx} = \sin\frac{1}{x} + x\left(\cos\frac{1}{x}\right)\left(-\frac{1}{x^2}\right)$ $\frac{dy}{dx} = \sin\frac{1}{x} - \frac{1}{x}\cos\frac{1}{x}$ $\frac{dy}{dx} = \sin\frac{\pi}{4} - \frac{\pi}{4}\cos\frac{\pi}{4}$ $= 0{\cdot}15$	Scale 15D (0, 4, 7, 11, 15) *Low Partial Credit* • any correct differentiation *Mid Partial Credit* • product rule applied *High Partial Credit* • correct differentiation **Note:** *one penalty for calculator in wrong mode*

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(b) Differentiate $x - \sqrt{x+6}$ with respect to x.

(c) Find the co-ordinates of the turning point of the function $y = x - \sqrt{x+6},\ x \geq -6$.

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(b) Differentiate $x-\sqrt{x+6}$ with respect to x.

$$f(x)=x-\sqrt{x+6}=x-(x+6)^{\frac{1}{2}}$$
$$f'(x)=1-\tfrac{1}{2}(x+6)^{-\frac{1}{2}}=1-\frac{1}{2\sqrt{x+6}}$$

(c) Find the co-ordinates of the turning point of the function $y=x-\sqrt{x+6},\ x\geq -6$.

$$f'(x)=0\Rightarrow 1-\frac{1}{2\sqrt{x+6}}=0$$
$$\Rightarrow 2\sqrt{x+6}=1$$
$$\Rightarrow x+6=\tfrac{1}{4}$$
$$\Rightarrow x=-5\tfrac{3}{4}$$
$$f(-5\tfrac{3}{4})=-5\tfrac{3}{4}-\sqrt{\tfrac{1}{4}}=-6\tfrac{1}{4}$$

$$\left(-5\tfrac{3}{4},-6\tfrac{1}{4}\right)$$