extra to the ordinary level material

Probability - Higher Level

Counting

Permutations formula

$${}^{n}P_{r} = \frac{n!}{(n-r)!}$$

but we usually use that

$${}^{n}P_{r} = \underbrace{n(n-1)(n-2)\cdots(n-(r-1))}_{\text{note: r terms}}$$

Combinations formula

${}^{n}C_{r}$

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

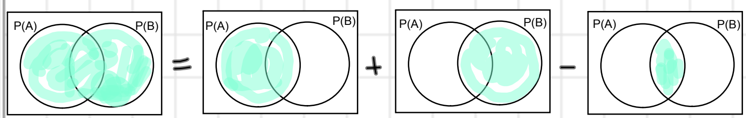
useful to know

$$\binom{n}{r} = \binom{n}{n-r} \quad \text{and} \quad \binom{n}{n} = \binom{n}{0} = 1$$

Probability

General Additional Rule

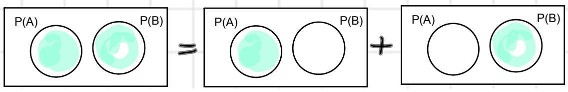
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$



If A and B are Mutually Exclusive

$\Rightarrow A \cap B = 0$

$$P(A \cup B) = P(A) + P(B)$$



Conditional Probability

$P(A|B)$ means probability that A happens given that B has already happened.

Conditional Probability Rule

$$P(A|B) = P(A \cap B) / P(B)$$

In general $P(A|B) \neq P(B|A)$

If A and B are not Independent

if $P(A) \neq P(A|B)$
then A is not independent B

If A and B are Independent

if $P(A) = P(A|B)$
then A is independent of B
also $\Rightarrow P(B|A) = P(B)$

General Multiplication Rule

$$P(A \cap B) = P(A) \times P(B|A)$$

Multiplication Rule
If A and B are Independent

$$P(A \cap B) = P(A) \times P(B)$$

To Show Independence

$P(A \cap B) = P(A) \times P(B) \Rightarrow$ A and B are independent

Bernoulli Trials (Binomial Distribution)

Probability of r successes in n Bernoulli trials is "n choose r" by the probability success (p) to the power of the number of successes (r) by the probability of failure (q) to the power of the number of failures (n−r).

Probability of k success in n Bernoulli trials

$$P(r) = \binom{n}{r} p^r q^{n-r}$$

Conditions for Bernoulli trials

- outcomes 'success' or 'failure'
- trials repeatable
- events independent
- fixed number of trials

Probability that the first success on nth Bernoulli trial

$$= P(n-1 \text{ failures}) \times P(1 \text{ success}) = p \cdot q^{n-1}$$

Problems involving up to 3 Bernoulli trials

eg solve .. $P(\text{success}, \text{success}, \text{failure}) = p \cdot p \cdot q$

or $P(\text{failure}, \text{failure}, \text{failure}) = q \cdot q \cdot p$

or $P(\text{success}, \text{success}, \text{success}) = p \cdot p \cdot p$

$$P(r) = \binom{n}{r} p^r q^{n-r}$$

$$(n-1) - (k-1) = n-k$$

Probability of kth success occurs on the nth Bernoulli trial

$$= P(k-1 \text{ successes in } n-1 \text{ trials}) \times P(\text{success})$$

$$= \binom{n-1}{k-1} p^{k-1} q^{n-k} \cdot p$$

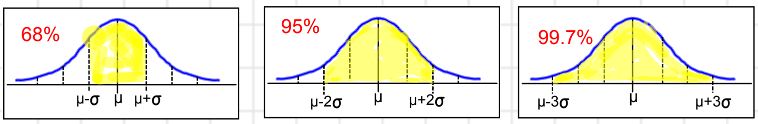
Probabilities from normal distribution curves

μ = 'mu'

σ = 'sigma'

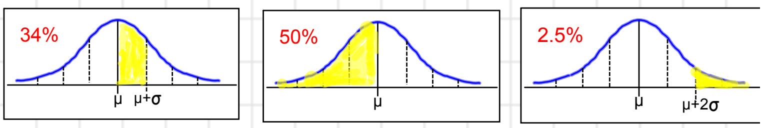
μ = mean σ = standard deviation

The empirical rule



We can use the empirical rule to work out other probabilities

eg.



z-score (standard score)

A z-score is the number of standard deviations that a value lies above or below the mean.

The standard normal distribution has a mean 0 and standard deviation 1.

$$z = \frac{x - \mu}{\sigma}$$

This formula changes given units into a z-score

Simulations in probability

what is a simulation why use a simulation?