Coordinate Geometry: The Line

chapter 1

Section 1.1 Revision of formulae

PROJECT MATHS – STRAND 2
Text & Tests 4
LEAVING CERTIFICATE HIGHER LEVEL

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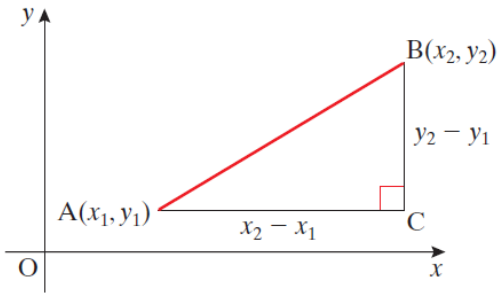
Distance between two points

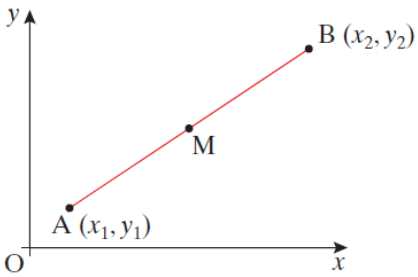
The distance between $A(x_1, y_1)$ and $B(x_2, y_2)$ is
$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$



2. The midpoint of a line segment

The midpoint M of the line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$ is

$$\left(\frac{x_1 + x_2}{2}, \quad \frac{y_1 + y_2}{2}\right)$$



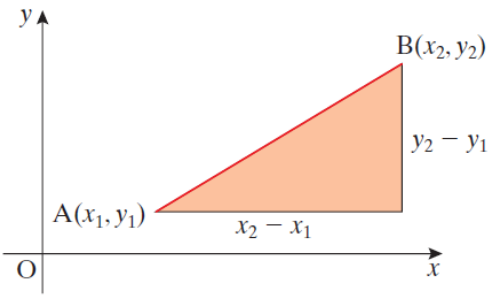
3. The slope of a line

In the diagram on the right, the slope, m, of AB is found by getting the value of

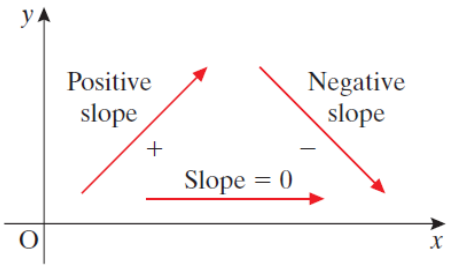
$$\frac{\text{vertical change}}{\text{horizontal change}} = \frac{y_2 - y_1}{x_2 - x_1}$$

The slope, m, of the line passing through (x_1, y_1) and (x_2, y_2) is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$



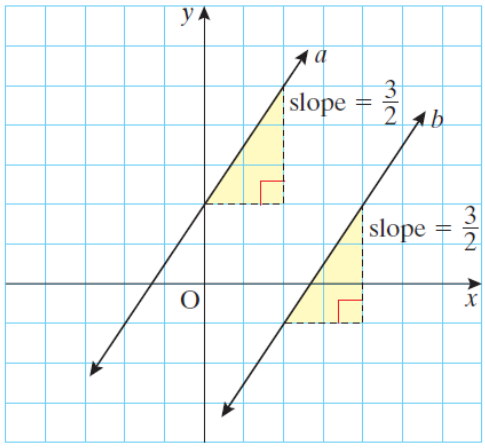
4. Positive and negative slopes



5. Parallel lines

The lines a and b in the given diagram both have the slope $\frac{3}{2}$.

Parallel lines have equal slopes.



6. Perpendicular lines

The given lines a and b are perpendicular.

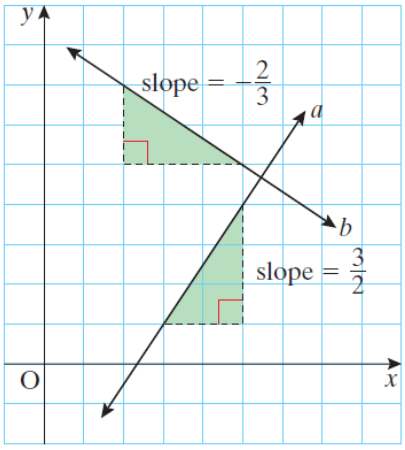
The slope of a is $\frac{3}{2}$.

The slope of $b = -\frac{2}{3}$.

Notice that one slope is the reciprocal of the other with the sign changed.

Notice also that the product of the two slopes is -1, i.e.,

$$-\tfrac{2}{3} \times \tfrac{3}{2} = -1$$



If two lines are perpendicular, the product of their slopes is -1, i.e.,

$$m_1 \times m_2 = -1$$

Example 1

A(3, 1), B(2, −3) and C(−1, k) are three points in the plane.
If AB ⊥ AC, find the value of k.

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m_{AB} \perp m_{AC}$

$m_{AB} = \frac{-3-1}{2-3} = \frac{-4}{-1} = 4$

$m_{AC} = \frac{k-1}{-1-3} = \frac{k-1}{-4}$

$4 \perp -\frac{1}{4}$

$\Rightarrow \frac{k-1}{-4} = -\frac{1}{4}$

$\frac{k-1}{4} = \frac{1}{4}$

$k - 1 = 1 \Rightarrow k = 2$

Exercise 1.1

1. Given three points A(−1, 3), B(3, −2) and C(5, 2).
Find (i) |AB| (ii) |BC| (iii) the slope of AC (iv) the midpoint of [BC].

$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

(i) $|AB| = \sqrt{(3--1)^2+(-2-3)^2} = \sqrt{(4)^2+(-5)^2}$
$= \sqrt{16+25} = \sqrt{41}$

(ii) $|BC| = \sqrt{(5-3)^2+(2--2)^2} = \sqrt{(2)^2+(4)^2}$
$= \sqrt{4+16} = \sqrt{20}$

$m = \frac{y_2-y_1}{x_2-x_1}$

(ii) $m_{AC} = \frac{2-3}{5--1} = \frac{-1}{6}$

midpt. = average point

(iv) $\text{midpt}_{[BC]} = \left(\frac{3+5}{2}, \frac{-2+2}{2}\right) = \left(\frac{8}{2}, \frac{0}{2}\right)$
$= (4, 0)$

2. Find M, the midpoint of the line segment joining A(1, −6) and B(−3, 4).
Hence show that |AM| = |MB|.

midpt = average pt.

$M = \left(\frac{1-3}{2}, \frac{-6+4}{2}\right) = \left(\frac{-2}{2}, \frac{-2}{2}\right) = (-1, -1)$

$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$|AM| = \sqrt{(-1-1)^2+(-1--6)^2} = \sqrt{(-2)^2+(5)^2}$
$= \sqrt{4+25} = \sqrt{29}$

$|MB| = \sqrt{(-3--1)^2+(4--1)^2} = \sqrt{(-2)^2+(5)^2}$
$= \sqrt{4+25} = \sqrt{29}$

$\Rightarrow |MB| = |BM|$

3. The slope of a line ℓ is $\frac{3}{4}$.

(i) What is the slope of any line parallel to ℓ?

(ii) What is the slope of any line perpendicular to ℓ?

parallel slopes are equal (i)

$\Rightarrow$ lines parallel to ℓ have slope $= \frac{3}{4}$

perpendicular slopes

$\frac{a}{b} \perp -\frac{b}{a}$

$\Rightarrow \frac{3}{4} \perp -\frac{4}{3}$

lines $\perp$ to ℓ have slope $= -4/3$