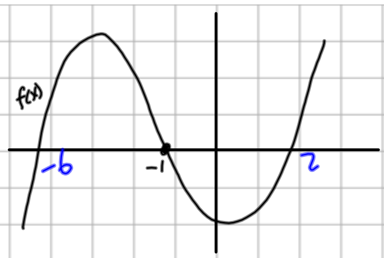
Mock Corrections
Paper 1

Q1

$f(x) = x^3 + 5x^2 + kx - 12$



Given $x+1$ is factor find k.

$\Rightarrow x = -1$ is solution $f(-1) = 0$

$f(-1) = (-1)^3 + 5(-1)^2 + k(-1) - 12$

$= -1 + 5 - k - 12 = 0$

$-8 - k = 0$

$k = -8$

Divide

$$\begin{array}{r l}
 & x^2 + 4x - 12 \\
x+1 \,\big)\!\! & \cancel{x^3} + 5x^2 - 8x - 12 \\
 & \mp \cancel{x^3} \mp x^2 \\
\hline
 & 4\cancel{x^2} - 8x \\
 & \mp 4\cancel{x^2} \mp 4x \\
\hline
 & -12x - 12 \\
 & \pm 12x \pm 12 \\
\hline
\end{array}$$

$x^2 + 4x - 12 = 0$

$(x+6)(x-2) = 0$

$\Rightarrow x = -6$

$x = 2$

Q2 $2-3i,\ 6+4i,\ -8+pi$ are consecutive terms in a geometric sequence. Find p.

$r = \frac{T_2}{T_1} = \frac{T_3}{T_2}$

$$\frac{6+4i}{2-3i} = \frac{-8+pi}{6+4i}$$

* by LCD $(2-3i)(6+4i)$

$(a+b)^2 = a^2+2ab+b^2$

$$(6+4i)^2 = (-8+pi)(2-3i)$$

$$36 + 48i \mp 16\cancel{i^2} = -16 + 24i + 2pi \pm 3p\cancel{i^2}$$

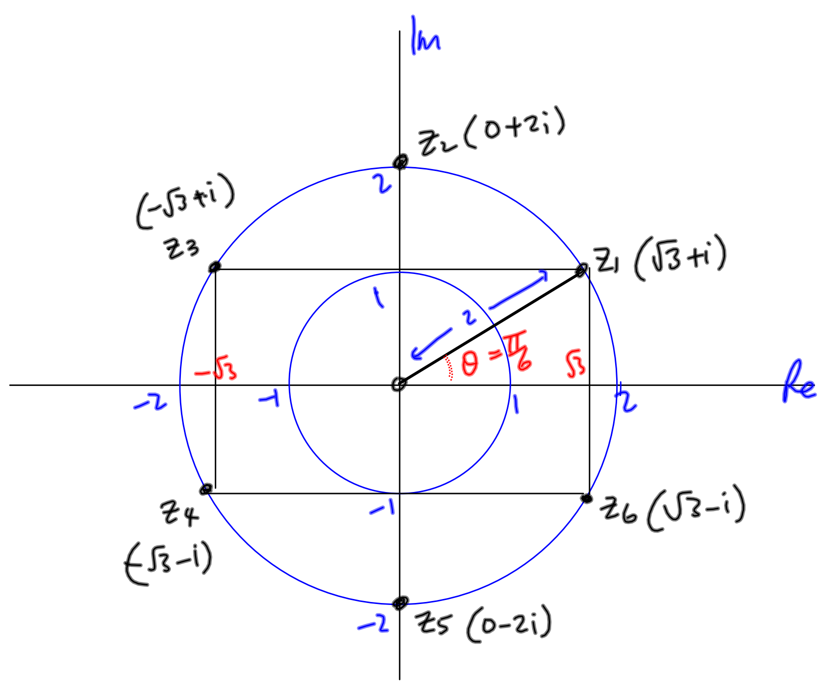
$$20 + 48i = (-16+3p) + (24+2p)i$$

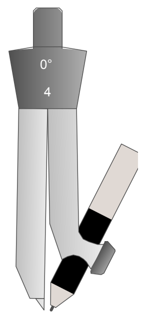
Re = Re, Im = Im

$20 = -16 + 3p$
$36 = 3p$
$p = 12$

$48 = 24 + 2p$
$24 = 2p$
$p = 12$

Q2 (b)





$$z_1 = \sqrt{3}+i = r\,\mathrm{Cis}\,\theta = 2\,\mathrm{Cis}\,\frac{\pi}{6} \quad \text{or} \quad 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$$

Q2 (b) $z^6 = a+bi$

Roots of z are:

$z_1 = \sqrt{3}+i$

$z_2 = 0+2i$

$\vdots$

de Moivre

$(r\,\mathrm{Cis}\,\theta)^n = r^n\,\mathrm{Cis}\,n\theta$

$\Rightarrow z_1^6 = a+bi$

$z_1 = 2\,\mathrm{Cis}\,\frac{\pi}{6}$

$$\begin{aligned} z_1^6 &= \left(2\,\mathrm{Cis}\,\tfrac{\pi}{6}\right)^6 \\ &= 2^6\,\mathrm{Cis}\,\tfrac{\cancel{6}\pi}{\cancel{6}} \\ &= 64(\cos\pi + i\sin\pi) \\ &= 64(-1 + i0) \\ &= -64 + 0i \end{aligned}$$

$\Rightarrow a = -64$

$b = 0$

Q3 Solve $\frac{x+3}{x-4} < 2$

$* (x-4)^2$

$$(x+3)(x-4) < 2(x-4)^2$$

$$x^2 - 4x + 3x - 12 < 2(x^2 - 8x + 16)$$

$$x^2 - x - 12 < 2x^2 - 16x + 32$$

$-2x^2, +16x, -32$

$$-x^2 + 15x - 44 < 0$$

$* -1$

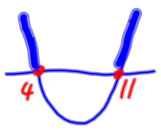
$$x^2 - 15x + 44 > 0$$

$$(x-11)(x-4) > 0$$

If $(x-11)(x-4) = 0$

$\Rightarrow x = 11$, $x = 4$ are solutions

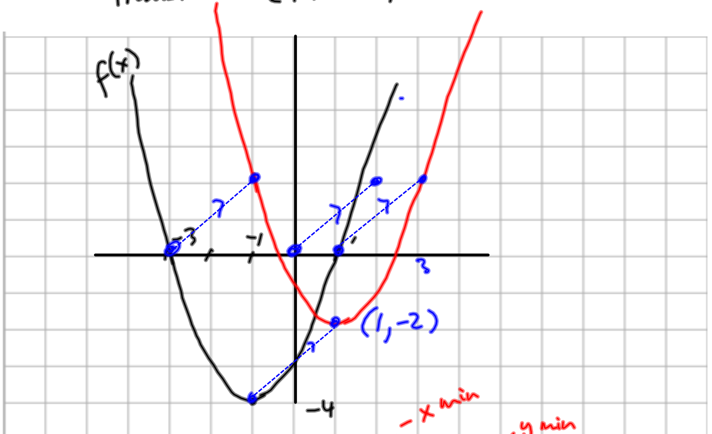
$4 > x > 11$



$\Rightarrow$ outside!

Q3 (b) $f(x) = x^2 + 2x - 3$

translation $(0,0) \to (2,2)$



- x min
y min

express in form $(x+a)^2 + b$

Complete square form

T&T6 Ch2

$g(x) = (x-1)^2 - 2$

Q3 b (iii) Solve $g(x) = (x-1)^2 - 2 = 0$

expand

$x^2 - 2x + 1 - 2 = 0$

$x^2 - 2x - 1 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

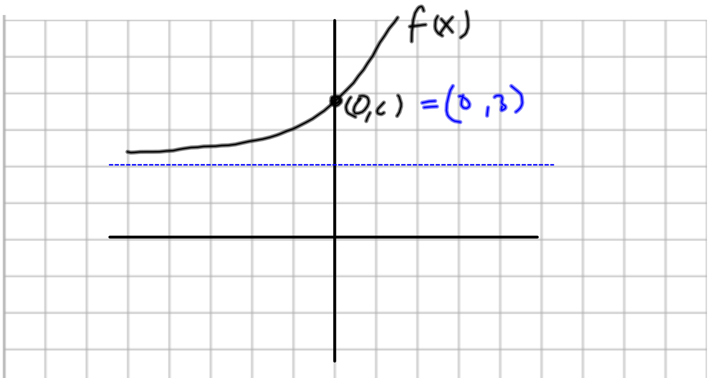
$x = \frac{+2 \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)}$

$= \frac{2 \pm \sqrt{4+4}}{2} = \frac{2 \pm 2\sqrt{2}}{2}$

$= 1 \pm \sqrt{2}$

Q4

(a) $f(x) = e^x + 2$

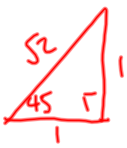


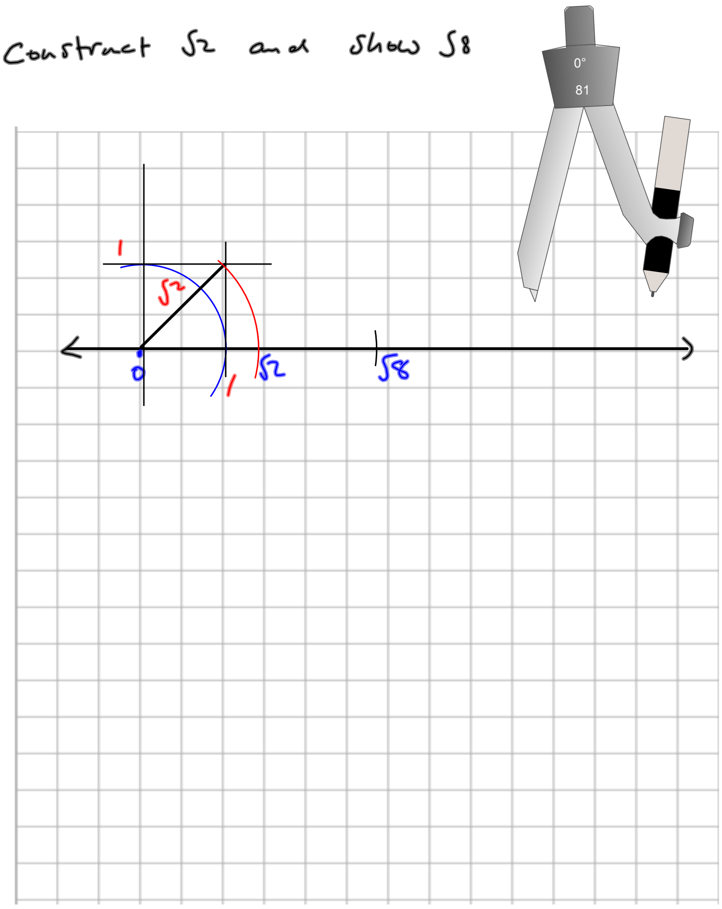
range > 2

$f(0) = e^0 + 2 = 1 + 2 = 3$

Q5

Construct $\sqrt{2}$ and show $\sqrt{8}$





$\sqrt{8} = 2\sqrt{2}$

Q5 (b) Arithmetic sequence $T_3 = 3\sqrt{2}+11$ $T_6 = 3\sqrt{8}+20$

(i) find a and d

$T_n = a+(n-1)d$

$\Rightarrow T_3 = a+(3-1)d = a+2d$

$\Rightarrow a+2d = 3\sqrt{2}+11$ (1)

$T_6 = a+(6-1)d = a+5d$

$a+5d = 3\sqrt{8}+20$ (2)

(2) − (1)

$\Rightarrow 0+3d = 3\sqrt{2}+9$

$d = \sqrt{2}+3$ (3)

(3) → (1)

$a+2(\sqrt{2}+3) = 3\sqrt{2}+11$

$a+2\sqrt{2}+6 = 3\sqrt{2}+11$

$a = \sqrt{2}+5$

Q5(b) ii Terms <100?

$T_n = a+(n-1)d$

$a = 5+\sqrt{2}$

$d = 3+\sqrt{2}$

$T_n = (5+\sqrt{2})+(n-1)(3+\sqrt{2})$

$(5+\sqrt{2})+(n-1)(3+\sqrt{2}) < 100$

$5+\cancel{\sqrt{2}}+3n+n\sqrt{2}-3-\cancel{\sqrt{2}} < 100$

$2+(3+\sqrt{2})n < 100$

$(3+\sqrt{2})n < 98$

$n < 98/(3+\sqrt{2})$

Multiply above and below by Conjugates

or use calculator!

$n < 42-14\sqrt{2} = 22.2$

$\Rightarrow$ 22 terms less than 100