

Exercise 7.3

1. Solve each of the following inequalities for $x \in R$.

(i) $|x + 3| = 1$

(ii) $|x - 2| = 4$

(iii) $|2x - 1| = 5$

Method 1:
Square both sides

(i) $|x + 3| = 1$
 $(x + 3)^2 = 1^2$
 $x^2 + 6x + 9 = 1$
 $x^2 + 6x + 8 = 0$
 $(x + 2)(x + 4) = 0$
 $x = -2$ or $x = -4$

Method 2:
Considering both options

(i) $|x + 3| = 1$
 \Rightarrow either $x + 3 = 1$ or $x + 3 = -1$
 $\Rightarrow x = 1 - 3$ or $x = -1 - 3$
 $x = -2$ or $x = -4$

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(i) $|x + 3| = 1$

(ii) $|x - 2| = 4$

(iii) $|2x - 1| = 5$

(iv) $|3x - 2| = x$

(v) $2|x - 3| = 2$

(vi) $|x - 5| = |x + 1|$

Square

$(a + b)^2$
 $= a^2 + 2ab + b^2$

(vi) $|x - 5| = |x + 1|$
 $(x - 5)^2 = (x + 1)^2$
 ~~$x^2 - 10x + 25 = x^2 + 2x + 1$~~
 $-10x - 2x = 1 - 25$
 $-12x = -24$
 $x = 2$

7. Solve the following inequalities, $x \in \mathbb{R}$.

(i) $|2x - 1| \geq 7$

(ii) $|3x + 4| \leq |x + 2|$

(iii) $2|x - 1| \leq |x + 3|$

Algebra

$(a+b)^2 = a^2 + 2ab + b^2$

Inside/outside?
0 is inside

(i)

$$(2x - 1)^2 \geq 7^2$$

$$4x^2 - 4x + 1 \geq 49$$

$$4x^2 - 4x - 48 \geq 0$$

$$x^2 - x - 12 \geq 0$$

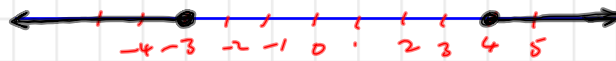
if $x^2 - x - 12 = 0$

$$(x - 4)(x + 3) = 0$$

$$x = 4 \text{ or } -3$$

test for $x = 0$, is $0^2 - 0 - 12 \geq 0$? No
 \Rightarrow outside values

$$-3 \geq x \geq 4$$



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(ii) $|3x + 4| \leq |x + 2|$

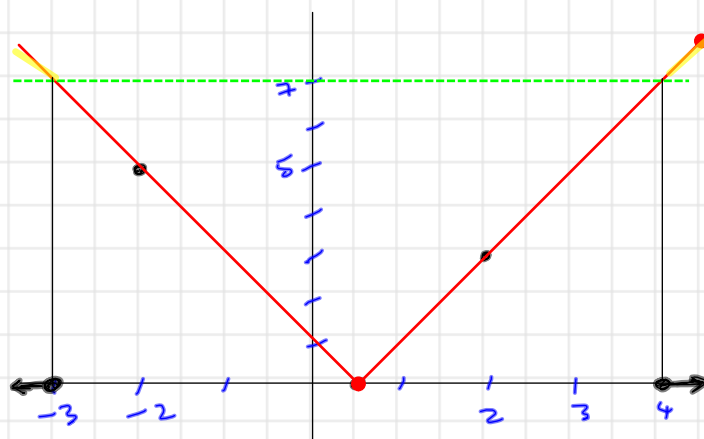
(iii) $2|x - 1| \leq |x + 3|$

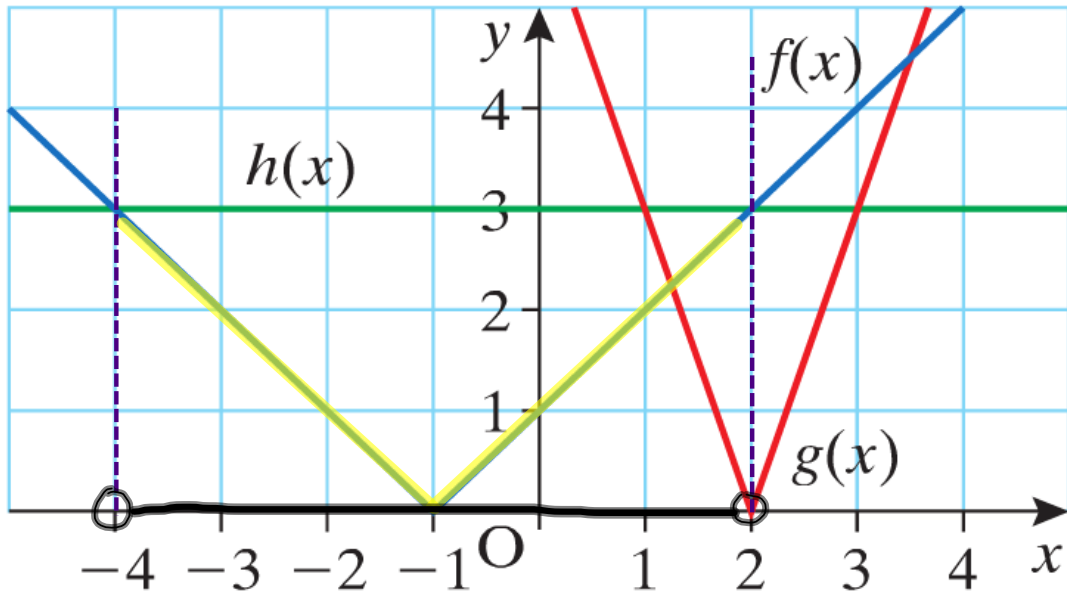
Graphing

(i)

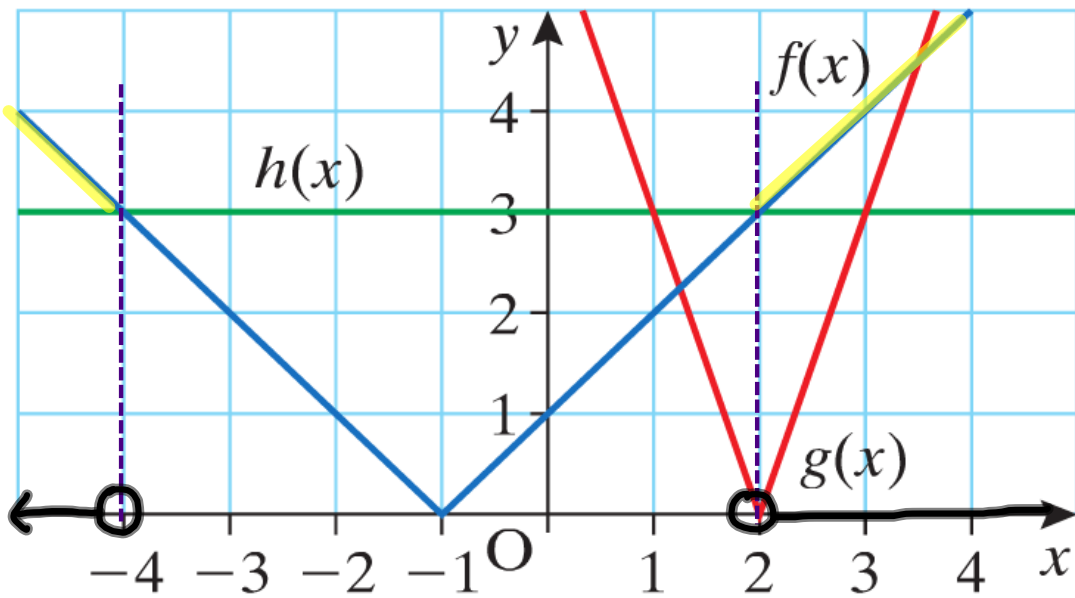
$$f(x) = |2x - 1|$$

x	f(x)
$\frac{1}{2}$	0
-2	5
+2	3

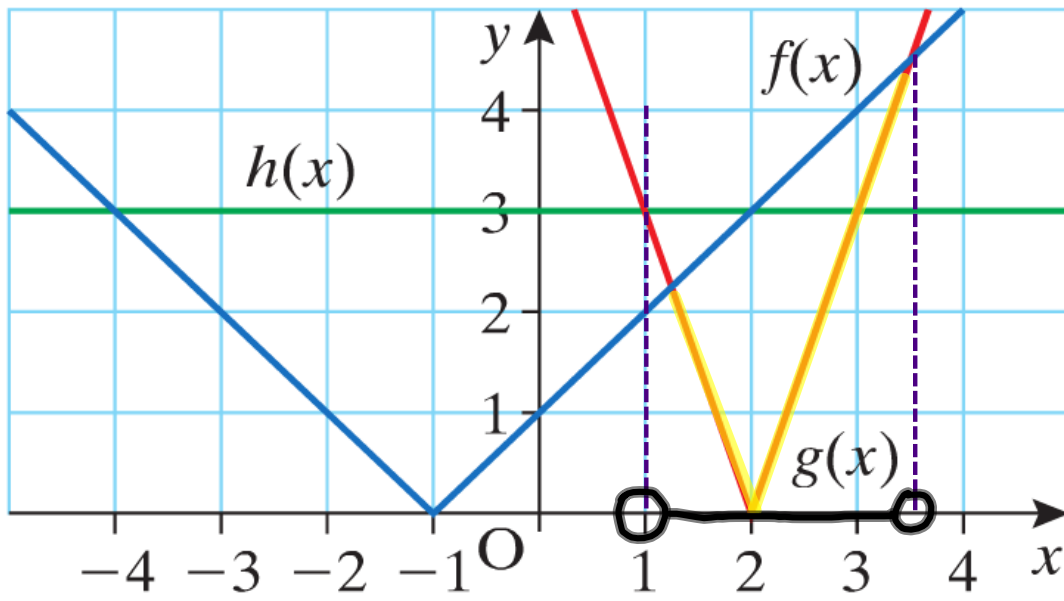




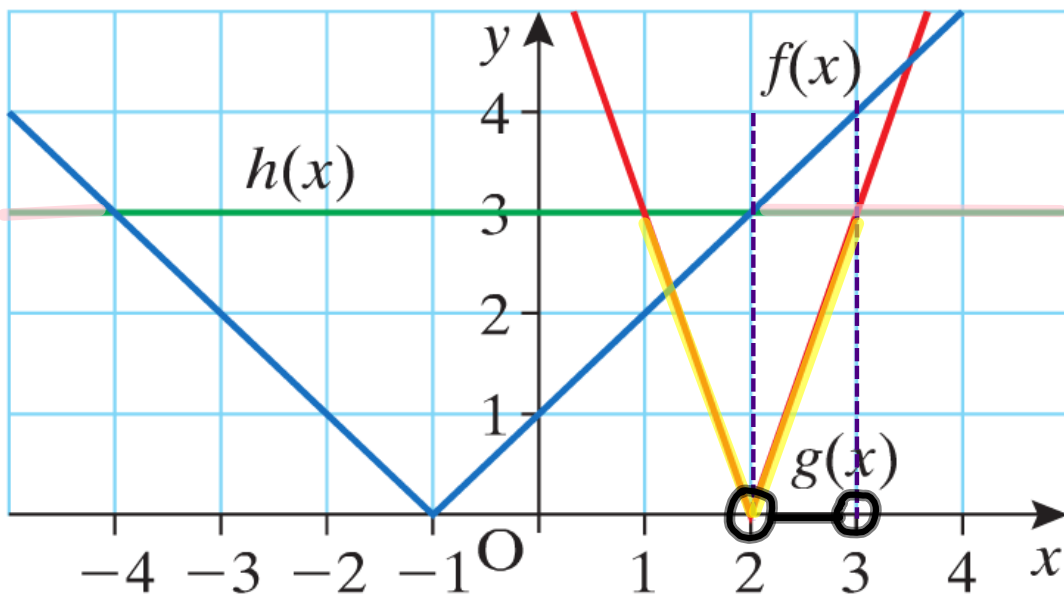
- (i) $f(x) < h(x)$ $-4 < x < 2$
- (ii) $h(x) < f(x)$
- (iii) $g(x) < f(x)$
- (iv) $g(x) < h(x) < f(x)$
- (v) $g(x) < f(x) < h(x)$
- (vi) $f(x) > h(x) > g(x)$
- (vii) $f(x) > g(x) > h(x)$



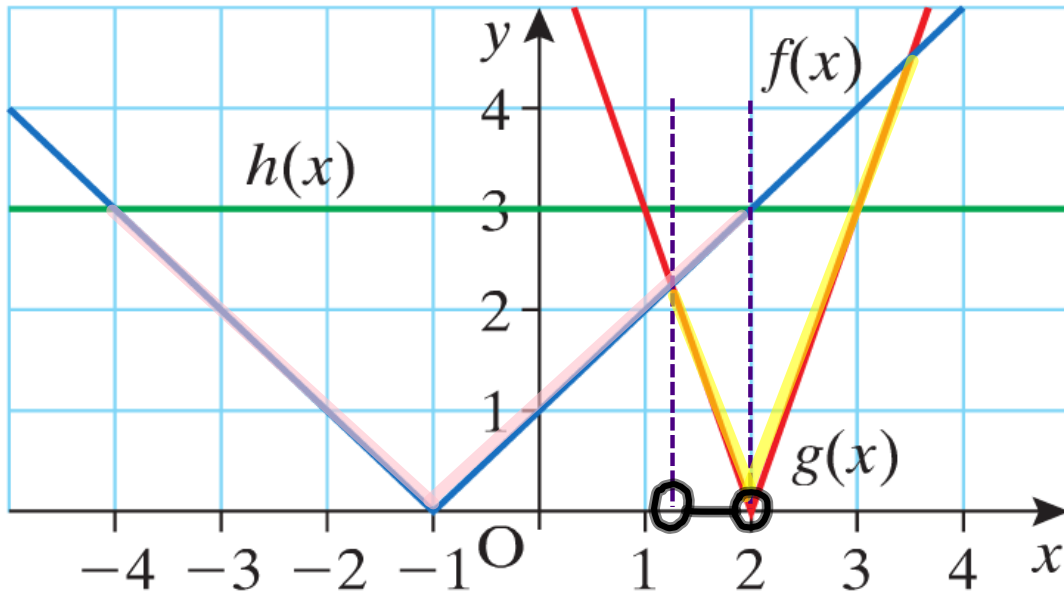
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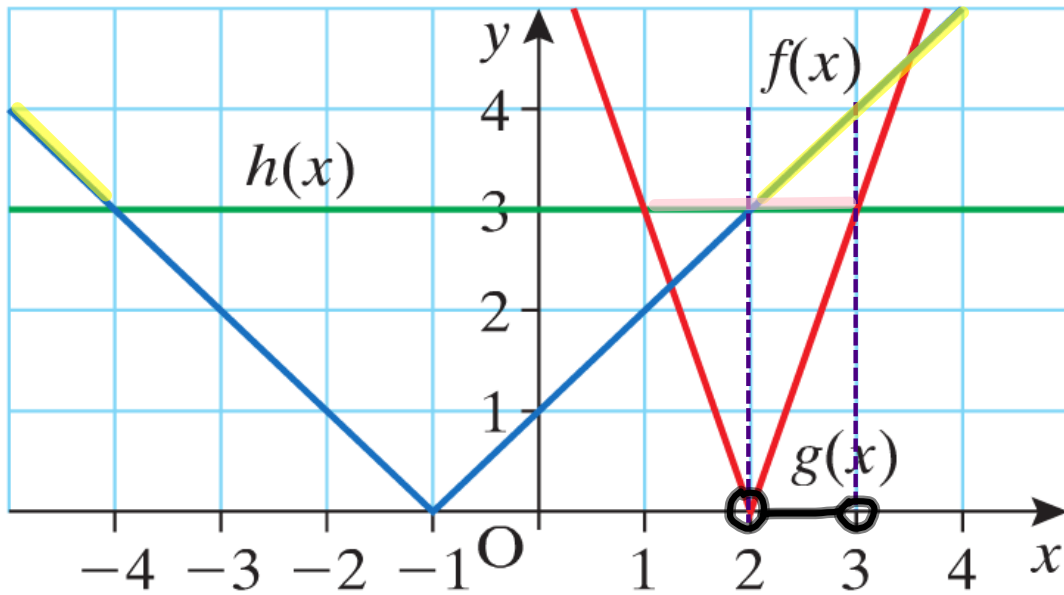
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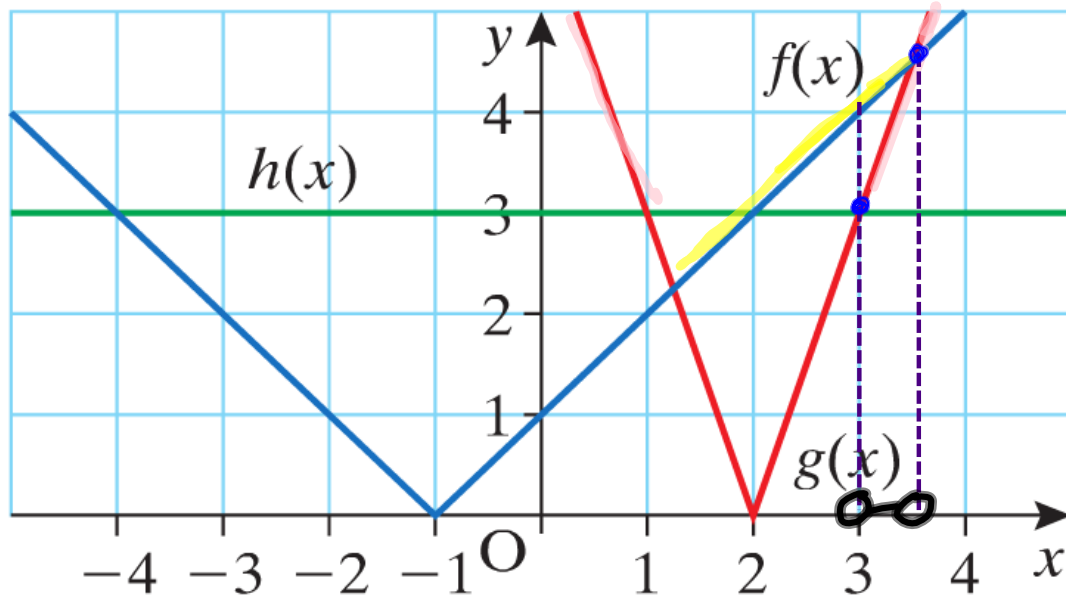
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- (iii) $g(x) < f(x)$ $1\frac{1}{4} < x < 3\frac{1}{2}$
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- (vii) $f(x) > g(x) > h(x)$ $3 < x < 3\frac{1}{2}$