Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate 2023

Marking Scheme

Mathematics

Higher Level

Note to teachers and students on the use of published marking schemes

Marking schemes published by the State Examinations Commission are not intended to be standalone documents. They are an essential resource for examiners who receive training in the correct interpretation and application of the scheme. This training involves, among other things, marking samples of student work and discussing the marks awarded, so as to clarify the correct application of the scheme. The work of examiners is subsequently monitored by Advising Examiners to ensure consistent and accurate application of the marking scheme. This process is overseen by the Chief Examiner, usually assisted by a Chief Advising Examiner. The Chief Examiner is the final authority regarding whether or not the marking scheme has been correctly applied to any piece of candidate work.

Marking schemes are working documents. While a draft marking scheme is prepared in advance of the examination, the scheme is not finalised until examiners have applied it to candidates' work and the feedback from all examiners has been collated and considered in light of the full range of responses of candidates, the overall level of difficulty of the examination and the need to maintain consistency in standards from year to year. This published document contains the finalised scheme, as it was applied to all candidates' work.

In the case of marking schemes that include model solutions or answers, it should be noted that these are not intended to be exhaustive. Variations and alternatives may also be acceptable. Examiners must consider all answers on their merits, and will have consulted with their Advising Examiners when in doubt.

Future Marking Schemes

Assumptions about future marking schemes on the basis of past schemes should be avoided. While the underlying assessment principles remain the same, the details of the marking of a particular type of question may change in the context of the contribution of that question to the overall examination in a given year. The Chief Examiner in any given year has the responsibility to determine how best to ensure the fair and accurate assessment of candidates' work and to ensure consistency in the standard of the assessment from year to year. Accordingly, aspects of the structure, detail and application of the marking scheme for a particular examination are subject to change from one year to the next without notice.

Coimisiún na Scrúduithe Stáit

State Examinations Commission

Leaving Certificate Examination 2023

Mathematics

Higher Level

Paper 1

Marking scheme

300 marks

Marking Scheme – Paper 1, Section A and Section B

Structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	A	B	C	D
No of categories	2	3	4	5
5 mark scales	0, 5	0, 2, 5	0, 2, 3, 5	0, 2, 3, 4, 5
10 mark scales	0, 10	0, 5, 10	0, 4, 7, 10	0, 3, 5, 8, 10
15 mark scales			0, 6, 12, 15	0, 4, 8, 12, 15
20 mark scales				0, 5, 10, 15, 20

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

Marking scales – level descriptors

A-scales (two categories)

- incorrect response
- correct response

B-scales (three categories)

- response of no substantial merit
- partially correct response
- correct response

C-scales (four categories)

- response of no substantial merit
- response with some merit
- almost correct response
- correct response

D-scales (five categories)

- response of no substantial merit
- response with some merit
- response about half-right
- almost correct response
- correct response

Section A

Question 1	**(30 marks)**
(a)	**15D**
(b)	**10D**
(c)	**5D**

Question 2	**(30 marks)**
(a)	**15D**
(b)	**5C**
(c)(i)(ii)	**10C**

Question 3	**(30 marks)**
(a)	**10D**
(b)(i)	**10D**
(b)(ii)	**10C**

Question 4	**(30 marks)**
(a)	**5C**
(b)	**15D**
(c)(i)(ii)(iii)	**10D**

Question 5	**(30 marks)**
(a)	**15C**
(b)	**5D**
(c)(i)(ii)	**10D**

Question 6	**(30 marks)**
(a)(i)	**10C**
(a)(ii)	**15D**
(b)	**5D**

Section B

Question 7	**(50 marks)**
(a)	**5B**
(b)	**10C**
(c)	**10C**
(d)	**10D**
(e)	**5C**
(f)	**5B**
(g)	**5D**

Question 8	**(50 marks)**
(a)	**5B**
(b)(i)(ii)	**10D**
(c)	**5C**
(d)(i)(ii)	**15D**
(e)	**10C**
(f)	**5B**

Question 9	**(50 marks)**
(a)(i)(ii)(iii)	**15D**
(b)(i)(ii)(iii)	**20D**
(c)(i)	**10D**
(c)(ii)	**5D**

Question 10	**(50 marks)**
(a)	**5B**
(b)	**10C**
(c)	**5C**
(d)	**10D**
e(i)	**10C**
e(ii)	**10D**

Palette of annotations available to examiners

Symbol	Name	Meaning in the body of the work	Meaning when used in the right margin
✓	Tick	Work of relevance	The work presented in the body of the script merits full credit
✗	Cross	Incorrect work (distinct from an error)	The work presented in the body of the script merits 0 credit
*	Star	Rounding / Unit / Arithmetic error / Misreading	
	Horizontal wavy	Error	
P	P		The work presented in the body of the script merits *Partial Credit*
L	L		The work presented in the body of the script merits *Low Partial Credit*
M	M		The work presented in the body of the script merits *Mid Partial Credit*
H	H		The work presented in the body of the script merits *High Partial Credit*
F*	F star		The work presented in the body of the script merits *Full Credit – 1*
[	Left Bracket		Another version of this solution is presented elsewhere and it merits equal or higher credit
	Vertical wavy	No work on this page / portion of this page	
O	Oversimplify	The candidate has oversimplified the work	
WOM	Work of merit	The candidate has produced work of merit (in line with that defined in the scheme)	
S	Stops early	The candidate has stopped early in this part	

Note: Where work of substance is presented in the body of the script, the annotation on the right margin should reflect a combination of annotations in the work.

In a **C scale** that is **not** marked using steps, where * and and appear in the body of the work, then L should be placed in the right margin.

In the case of a **D scale** with the same annotations, M should be placed in the right margin.

Detailed marking notes

Model Solutions & Marking Notes

Note: The model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any Examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her Advising Examiner.

<table>
<tr><th>Q1</th><th>Model Solution –30 Marks</th><th>Marking Notes</th></tr>
<tr><td>(a)</td><td>

Method 1:

$5 + 3m = 11$ $5 + 3m = -11$
$3m = 6$ $3m = -16$
$m = 2$ $m = -\frac{16}{3}$

OR

Method 2:
$(5 + 3m)^2 = 11^2$
$25 + 30m + 9m^2 = 121$
$9m^2 + 30m - 96 = 0$
$3m^2 + 10m - 32 = 0$
$(3m + 16)(m - 2) = 0$
$m = -\frac{16}{3},\ \ m = 2$

</td><td>

Scale 15D (0, 4, 8, 12, 15)
Method 1
Low Partial Credit:
- 1 linear equation.
- One correct value of m found without work.
- Attempts at trial and improvement.

Mid Partial Credit:
- One value of m found with work.

High Partial Credit
- One value of m correctly found and work of merit in finding the second value.

Method 2
Note: If quadratic does not have an m term award Mid Partial Credit at most

Low Partial Credit:
- Indication of squaring

Mid Partial Credit:
- relevant quadratic in m expanded (Line 2 of the solution)
- Quadratic is missing the m term, otherwise correct

High Partial Credit
- quadratic factorised
- quadratic formula fully substituted

</td></tr>
</table>

Q1	Model Solution –30 Marks	Marking Notes
(b)	$j + k = hk$ $k - hk = -j$ $k(1-h) = -j$ $k = -\frac{j}{1-h}$ or $k = \frac{j}{h-1}$	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* • Work of merit in eliminating fractions *Mid Partial Credit* • Terms with k transposed to one side of the equation *High Partial Credit* • $k(1-h) = -j$ or equivalent

<table>
<tr><th>Q1</th><th>Model Solution –30 Marks</th><th>Marking Notes</th></tr>
<tr>
<td>(c)</td>
<td>

Method 1

$$\begin{array}{r} x+p \\ x^2-px+1\overline{)x^3+0x^2-2x-3r} \\ \underline{x^3-px^2+x} \\ px^2-3x-3r \\ \underline{px^2-p^2x+p} \\ (p^2-3)x-3r-p \end{array}$$

$p^2-3=0$
$p^2=3$
$p=-\sqrt{3}\ [p<0]$

$-3r-p=0$
$-3r-\left(-\sqrt{3}\right)=0$
$\sqrt{3}=3r$
$r=\frac{\sqrt{3}}{3}$

OR

Method 2

$(x^2-px+1)(x+k)=x^3-2x-3r$

$x^3-px^2+x+kx^2-pkx+k=x^3-2x-3r$

$x^3+(k-p)x^2+(1-pk)x+k=x^3-2x-3r$

$k-p=0$
$k=p$

$1-pk=-2$
$1-p^2=-2$
$p^2=3$
$p=-\sqrt{3}\ \ [p<0]$

$k=-3r$
$r=-\frac{k}{3}$
$=-\frac{-\sqrt{3}}{3}$
$=\frac{\sqrt{3}}{3}$

OR

Method 3

	x	$-3r$
x^2	x^3	$-3rx^2$
$-px$	$-px^2$	$+3rpx$
$+1$	x	$-3r$

$-p-3r=0$
$p=-3r$
$r=-\frac{p}{3}$

$1+3rp=-2$
$rp=-1$
$-\frac{p}{3}(p)=-1$
$p^2=3$
$p=-\sqrt{3}\ \ [p<0]$
$r=\frac{\sqrt{3}}{3}$

</td>
<td>

Scale 5D (0, 2, 3, 4, 5)

Note: Full credit -1 if $\boldsymbol{p}=\sqrt{\mathbf{3}}$ but otherwise correct

Method 1
4 steps:
1. Sets up long division
2. First cycle in long division correct
3. Value of $\boldsymbol{p}$ found
4. Value of $\boldsymbol{r}$ found

Low Partial Credit:
Work of merit, for example, some correct division, or sets up long division.

Mid Partial Credit:
- 2 steps correct

High Partial Credit
- 3 steps correct

Method 2
4 steps:
1. Equation set up
2. Expansion of the product (Allow with 3 or more terms correct)
3. Value of p found
4. Value of r found

Low Partial Credit:
- Work of merit, for example, mentions linear factor

Mid Partial Credit:
- 2 steps correct

High Partial Credit
- 3 steps correct.

Method 3
1. Grid set up
2. Grid completed (Allow with 3 or more terms correct)
3. Value of p found
4. Value of r found

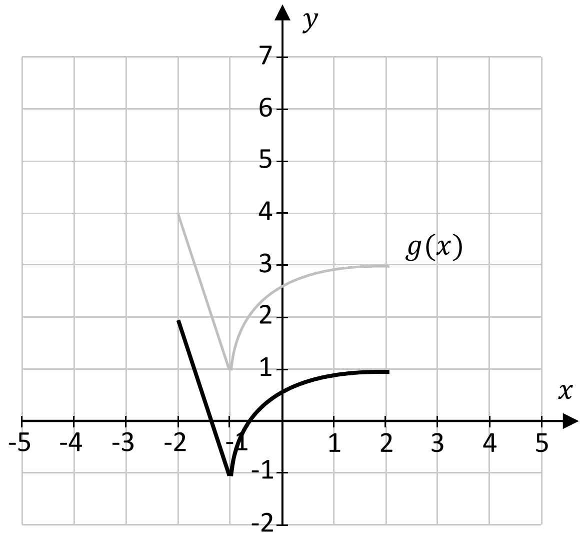
Low Partial Credit:
- Work of merit, for example, mentions linear factor.

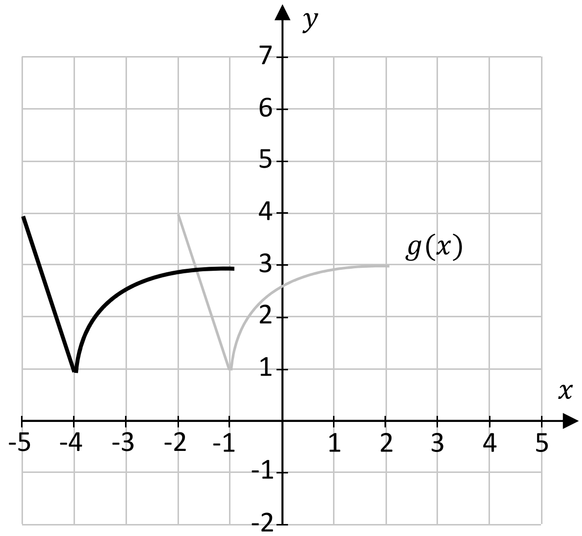
Mid Partial Credit:
- 2 steps correct.

High Partial Credit
- 3 steps correct.

</td>
</tr>
</table>

Q2	Model Solution – 30 Marks	Marking Notes
(a)	$f'(x) = 2x + b$ $f'(3) = 2(3) + b = 0$ $b = -6$ $f(3) = (3)^2 - 6(3) + c = -1$ $9 - 18 + c = -1$ $c = 8$ **OR** $x^2 + bx + c = \left(x + \frac{b}{2}\right)^2 - \frac{b^2}{4} + c$ $-\frac{b}{2} = 3$ so $b = -6$ $-\frac{b^2}{4} + c = -1$ so $c = 8$ **OR** $f(x) = (x - 3)^2 - 1$ $= x^2 - 6x + 8$	**Scale 15D (0, 4, 8, 12, 15)** *Low Partial Credit:* • Work of merit, for example, $f(3)$ or some correct differentiation • Work of merit at completing the square • $(x - h)^2 + k$ *Mid Partial Credit:* • b correct • Uses $f(3)$ to find a correct equation in b and c • $\left(x + \frac{b}{2}\right)^2 - \frac{b^2}{4} + c$ • Work of merit in finding both b and c • $(x - 3)^2 + k$, where $k \neq -1$ • $(x - h)^2 - 1$, where $h \neq 3$ *High Partial Credit* • Finds b and work of merit in finding c • $(x - 3)^2 - 1$
(b)	$\lim_{n\to\infty}\left(\frac{n}{n+1} + \frac{n+1000}{n} + \left(\frac{1}{3}\right)^n\right)$ $= \lim_{n\to\infty}\left(\frac{n}{n+1}\right) + \lim_{n\to\infty}\left(\frac{n+1000}{n}\right) + \lim_{n\to\infty}\left(\left(\frac{1}{3}\right)^n\right)$ $\left[= \lim_{n\to\infty}\left(\frac{1}{1+\frac{1}{n}}\right) + \lim_{n\to\infty}\left(\frac{1+\frac{1000}{n}}{1}\right) + \lim_{n\to\infty}\left(\left(\frac{1}{3}\right)^n\right)\right]$ $= \frac{1}{1+0} + \frac{1+0}{1} + 0$ $= 2$	**Scale 5C (0, 2, 3, 5)** Note: Full credit for correct answer without work. *Low Partial Credit:* • Work of merit, for example, indicates sum of limits, divides by highest power of n in one of first two terms • Substitutes ∞ for n • Finds two or more terms of the sequence, $T_n = \frac{n}{n+1} + \frac{n+1000}{n} + \left(\frac{1}{3}\right)^n$ *High Partial Credit:* • One limit correctly evaluated and work of merit in any one of the other two limits

<table>
<tr><th>Q2</th><th>Model Solution – 30 Marks</th><th>Marking Notes</th></tr>
<tr><td>(c)</td><td>

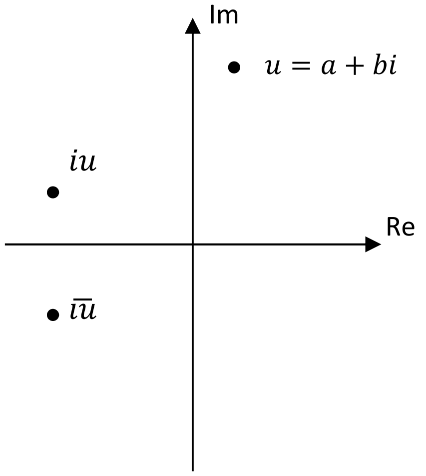

</td><td>Scale 10C (0, 4, 7, 10)
Low Partial Credit:
Work of merit in one part, for example, one point correctly transformed
In part (i) any vertical translation of $g(x)$
In part (ii) any horizontal translation of $g(x)$
In part (i) finds $g(x+2)$ or $g(x-2)$
In part (ii) finds $g(x)-3$ or $g(x)+3$
High Partial Credit:
One part correct
Work of merit in both parts</td></tr>
</table>

Q3	Model Solution – 30 Marks	Marking Notes
(a)	Assume that $\sqrt{2}$ is rational. $\sqrt{2} = \frac{a}{b}$ where $a, b \in \mathbb{Z}, b \neq 0$ and $\text{HCF}(a\,, b) = 1$ $2 = \frac{a^2}{b^2}$ $2b^2 = a^2$ $\Rightarrow a^2$ is even If a^2 is even, then a is even. $\therefore a = 2k$, where $k \in \mathbb{Z}$ $2b^2 = (2k)^2$ $2b^2 = 4k^2$ $b^2 = 2k^2$ $\therefore b^2$ is even If b^2 is even, then b is even. If both a and b are even, then they have 2 as a common factor. This contradicts the assumption that $\text{HCF}(a, b) = 1$.	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* • Work of merit, for example $\sqrt{2} = \frac{a}{b}$ • Work of merit in showing that a is even *Mid Partial Credit:* • Shows that a is even *High Partial Credit* • Shows both a and b are even
(b)	**Method 1** $\log_3 t + \frac{\log_3 t}{\log_3 9} + \frac{\log_3 t}{\log_3 27} + \frac{\log_3 t}{\log_3 81} = 10$ $\log_3 t + \frac{\log_3 t}{2} + \frac{\log_3 t}{3} + \frac{\log_3 t}{4} = 10$ $12\log_3 t + 6\log_3 t + 4\log_3 t + 3\log_3 t = 120$ $25\log_3 t = 120$ $\log_3 t = \frac{120}{25}$ $t = 3^{\frac{120}{25}} = 3^{\frac{24}{5}}$ **OR** **Method 2** $\frac{1}{\log_t 3} + \frac{1}{\log_t 9} + \frac{1}{\log_t 27} + \frac{1}{\log_t 81} = 10$ $\frac{1}{\log_t 3} + \frac{1}{2\log_t 3} + \frac{1}{3\log_t 3} + \frac{1}{4\log_t 3} = 10$ $\frac{25}{12\log_t 3} = 10$ $\log_t 3 = \frac{25}{120}$ $t^{\frac{25}{120}} = 3$ $t = 3^{\frac{120}{25}}$	**Scale 10D (0, 3, 5, 8, 10)** **3 steps:** **1.** Changing all to the same base **2.** Simplifies to an equation in $\boldsymbol{t}$ with one log **3.** Finds $\boldsymbol{t}$ *Low Partial Credit:* • Work of merit, for example, changes the base of one log (from the given equation) • Writes either 9, 27 or 81 in the form 3^k *Mid Partial Credit:* • One correct step *High Partial Credit* • 2 correct steps

Q3	Model Solution – 30 Marks	Marking Notes
(c) **(i)** **(ii)**	**(i)** *Any valid explanation, for example*: the power you need to raise 6 to, to get m. **(ii)** $\log_6 m > 1$	**Scale 10C (0, 4, 7, 10)** Note: Accept $\boldsymbol{6^x = m}$ as a valid explanation for (i) *Low Partial Credit:* • Work of merit in (i) or (ii), for example, some reference to indices • $\log_6 m > 0$ or $\log_6 m$ is positive *High Partial Credit* • (i) or (ii) correct

Q4	Model Solution – 30 Marks	Marking Notes
(a)	**Method 1** $(1+i)^2+(3-2i)(1+i)+p=0$ $1+2i+i^2+3+i-2(i)^2+p=0$ $5+3i+p=0$ $p=-5-3i$ **Method 2** Let the second root $= z_2$ Sum of roots: $1+i+z_2=-3+2i$ $z_2=-4+i$ Product of roots: $(1+i)(-4+i)=p$ $p=-5-3i$ **Method 3** $z=\frac{-(3-2i)\pm\sqrt{(3-2i)^2-4p}}{2}$ $2z=-(3-2i)\pm\sqrt{(3-2i)^2-4p}$ $2z+3-2i=\pm\sqrt{(3-2i)^2-4p}$ $[2z+3-2i]^2=(3-2i)^2-4p$ $z=1+i$ satisfies this equation $[2(1+i)+3-2i]^2=(3-2i)^2-4p$ $5^2=(3-2i)^2-4p$ $4p=-20-12i$ $p=\frac{-20-12i}{4}$ $=-5-3i$ **Method 4** $z+(4-i)$ $z-1-i/\overline{z^2+(3-2i)z+p}$ $\underline{z^2-(1+i)z}$ $(4-i)z+p$ $\underline{(4-i)z-5-3i}$ $p+5+3i$ $p+5+3i=0$ $p=-5-3i$	**Scale 5C (0, 2, 3, 5)** Note: Any attempt involving the conjugate of $\mathbf{1+i}$, award Low Partial Credit at most. **Method 1** *Low Partial Credit:* • Work of merit, for example, some correct substitution or some correct multiplication *High Partial Credit*: • Fully correct substitution and multiplication **Method 2** *Low Partial Credit:* • $z^2-(\text{sum})z+\text{product}$ • States p is the product of the roots • Sum of the roots = $-3+2i$ *High Partial Credit:* • Finds 2nd root • States sum of the roots = $3-2i$, but finishes correctly **Method 3** *Low Partial Credit:* • Some correct substitution in the quadratic formula *High Partial Credit:* • Formula fully substituted and $1+i$ substituted for z • Formula fully substituted and set equal to $1+i$ **Method 4** *Low Partial Credit:* Sets up long division but divisor must be of the form $z-a+bi$, where $a=1$ and $b=-1$ (Accept $b=1$ here) *High Partial Credit:* • First cycle of long division done correctly.

Q4	Model Solution – 30 Marks	Marking Notes
(b)	<u>Reference Angle:</u> $\alpha = \tan^{-1}\frac{\sqrt{3}}{1} = 60° \ \left(\frac{\pi}{3}\text{ rads}\right)$ <u>Argument:</u> $\theta = 180^0 - 60^0 = 120^0\left(\frac{2\pi}{3}\text{ rads}\right)$ <u>Modulus:</u> $r = \sqrt{(-1)^2 + \left(\sqrt{3}\right)^2}$ $= \sqrt{4}$ $= 2$ <u>General Polar Form:</u> $2\left(\cos\left(\frac{2\pi}{3} + 2n\pi\right) + i\sin\left(\frac{2\pi}{3} + 2n\pi\right)\right)$ $w^2 = 2\left(\cos\left(\frac{2\pi}{3} + 2n\pi\right) + i\sin\left(\frac{2\pi}{3} + 2n\pi\right)\right)$ $w = \left[2\left(\cos\left(\frac{2\pi}{3} + 2n\pi\right) + i\sin\left(\frac{2\pi}{3} + 2n\pi\right)\right)\right]^{\frac{1}{2}}$ <u>De Moivre:</u> $w = 2^{\frac{1}{2}}\left[\left(\cos\frac{1}{2}\left(\frac{2\pi}{3} + 2n\pi\right) + i\sin\frac{1}{2}\left(\frac{2\pi}{3} + 2n\pi\right)\right)\right]$ $= 2^{\frac{1}{2}}\left[\cos\left(\frac{\pi}{3} + n\pi\right) + i\sin\left(\frac{\pi}{3} + n\pi\right)\right]$ <u>$n = 0$:</u> $w = \sqrt{2}\left(\cos\left(\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right)\right)$ $= \sqrt{2}\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$ $= \frac{\sqrt{2}}{2} + \frac{\sqrt{6}}{2}i$ <u>$n = 1$:</u> $w = \sqrt{2}\left(\cos\left(\frac{\pi}{3} + \pi\right) + i\sin\left(\frac{\pi}{3} + \pi\right)\right)$ $= \sqrt{2}\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)$ $= -\frac{\sqrt{2}}{2} - \frac{\sqrt{6}}{2}i$	**Scale 15D (0, 4, 8, 12, 15)** Note: polar form must be used to achieve any credit. Note: Accept correct polar form without work (i.e., finding $\boldsymbol{r}$ and $\boldsymbol{\theta}$) Note: if $(\boldsymbol{w}^2)^2$ is found, award *Mid Partial Credit* at most. Note: general polar form is not required to find the roots. Note: Accept solution in decimal form. **4 steps:** **1**. Finds θ **2**. Finds r **3.** One root evaluated from De Moivre's expression **4.** 2nd root found *Low Partial Credit:* • Work of merit, for example, plots $-1 + \sqrt{3}i$ • Work of merit towards finding r or θ • $w = \left(-1 + \sqrt{3}i\right)^{\frac{1}{2}}$ *Mid Partial Credit* • 2 steps correct *High Partial Credit* • 3 steps correct *Full Credit –1* • Roots found correctly, but one or both in polar form

Q4	Model Solution – 30 Marks	Marking Notes
(c) **(i)** **(ii)** **(iii)**	**(i)** $iu = i(a+bi)$ $= ai + bi^2$ $= -b + ai$ $\overline{iu} = -b - ai$ **(ii)**  **(iii)** 90° counterclockwise rotation about the origin followed by axial symmetry in Re axis (x axis) Axial symmetry in the Im axis (y axis) followed by a 90° counterclockwise rotation about the origin. Axial symmetry in a line through the origin with slope -1 or similar	**Scale 10D (0, 3, 5, 8, 10)** Note: 4 elements required: iu, $\overline{iu}$, plot, transformation Note: Accept conjugate plot from either candidate's work in (i) or by reflection of their $\overline{iu}$ in the real axis *Low Partial Credit:* • Work of merit in one element, for example, $i(a+bi)$ *Mid Partial Credit* • 1 element correct **and** work of merit in a 2nd element *High Partial Credit* • 2 elements correct **and** work of merit in a 3rd element *Full Credit -1* • Diagram not labelled, otherwise correct

<table>
<tr><th>Q5</th><th>Model Solution – 30 Marks</th><th>Marking Notes</th></tr>
<tr><td>(a)</td><td>

$f(x) = (5x^2 + 7)^{-1}$

$f'(x) = -1(5x^2 + 7)^{-2}(10x)$

$= -10x(5x^2 + 7)^{-2}$

$f'(x) = \dfrac{-10x}{(5x^2 + 7)^2}$

OR

$u(x) = 1$ so $u'(x) = 0$

$v(x) = 5x^2 + 7$ so $v'(x) = 10x$

$f'(x) = \dfrac{(5x^2+7)(0) - 1(10x)}{(5x^2+7)^2}$

$= \dfrac{-10x}{(5x^2+7)^2}$

</td><td>

Scale 15C (0, 6, 12, 15)

Accept $-10x(5x^2 + 7)^{-2}$ for full credit.

Low Partial Credit:

- Some correct differentiation

High Partial Credit:

- Correct substitution into quotient rule
- One error in substitution into quotient rule, but finishes correctly

Full Credit –1

- $f'(x) = -1(5x^2 + 7)^{-2}(10x)$

</td></tr>
<tr><td>(b)</td><td>

$u = \tan\frac{x}{2}$ $\quad v = \ln x$

$\frac{du}{dx} = \frac{1}{2}\sec^2\frac{x}{2}$ $\quad \frac{dv}{dx} = \frac{1}{x}$

$g'(x) = \left(\tan\frac{x}{2}\right)\left(\frac{1}{x}\right) + (\ln x)\left(\frac{1}{2}\sec^2\frac{x}{2}\right)$

At $x = \frac{\pi}{2}$:

$g'(x) = \left(\tan\frac{\left(\frac{\pi}{2}\right)}{2}\right)\left(\frac{1}{\left(\frac{\pi}{2}\right)}\right) + (\ln\frac{\pi}{2})\left(\frac{1}{2}\sec^2\frac{\left(\frac{\pi}{2}\right)}{2}\right)$

$= 1\left(\frac{2}{\pi}\right) + \ln\frac{\pi}{2}\left(\frac{1}{2}(2)\right)$

$= \frac{2}{\pi} + \ln\frac{\pi}{2}$

</td><td>

Scale 5D (0, 2, 3, 4, 5)

No differentiation no credit

4 steps

1. Finds $\frac{du}{dx}$

2. Finds $\frac{dv}{dx}$

3. Applies the product rule correctly

4. Evaluates at $x = \frac{\pi}{2}$

Low Partial Credit:

- Any correct differentiation

Mid Partial Credit

- 2 steps correct

High Partial Credit

- 3 steps correct

Full Credit –1

- Answer not in the correct form

</td></tr>
</table>

Q5	Model Solution – 30 Marks	Marking Notes
(c) **(i)** **(ii)**	**(i)** $g\big(f(3)\big) = g($🌍$) = w$ **(ii) Injective**: no element of *C* is used more than once **or** no two elements in B go to the same element, **or any other valid reason.** **Not surjective**: one element of *C* is not used, **or** Range $\neq$ Codomain, **or** $\#B < \#C$, or **any other valid reason.**	**Scale 10D (0, 3, 5, 8, 10)** Three parts to check: (i) and injective and not surjective. In part (i) accept $g\big(f(3)\big)$, where g and f are the functions from part (b). *Low Partial Credit:* • Shows some understanding of injective **or** surjective functions. • Work of merit in any part, for example, $f(3)$ correct, merit in explanation of injective or surjective (e.g., in injective, $\#B \leq \#C$) *Mid Partial Credit* • One of the 3 parts correct *High Partial Credit* • Two parts correct

Q6	Model Solution – 30 Marks	Marking Notes
(a)(i)	$x + 4 = x^2 - 2$ $x^2 - x - 6 = 0$ $(x - 3)(x + 2) = 0$ $x = 3, \qquad x = -2$	**Scale 10C (0, 4, 7, 10)** *Low Partial Credit:* • Work of merit, for example, equation correctly established *High Partial Credit:* • Factors correct • Quadratic formula fully substituted • One correct answer verified

<table>
<tr><th>Q6</th><th>Model Solution – 30 Marks</th><th>Marking Notes</th></tr>
<tr><td>(a)(ii)</td><td>

Method 1

$$f(x) - g(x) = x + 4 - (x^2 - 2)$$
$$= -x^2 + x + 6$$
$$\text{Area} = \int_{-1}^{2} (-x^2 + x + 6)\, dx$$
$$= \left[-\frac{x^3}{3} + \frac{x^2}{2} + 6x\right]_{-1}^{2}$$

$$= \left(-\frac{(2)^3}{3} + \frac{2^2}{2} + 6(2)\right) - \left(-\frac{(-1)^3}{3} + \frac{(-1)^2}{2} + 6(-1)\right)$$

$$= \frac{33}{2}\ [\text{units}^2]$$

OR

Method 2

$$\text{Area}_1 = \int_{-1}^{2} (x + 4)\, dx$$
$$= \left[\frac{x^2}{2} + 4x\right]_{-1}^{2}$$
$$= \left[\left(\frac{(2)^2}{2} + (8)\right) - \left(\frac{(-1)^2}{2} + (-4)\right)\right]$$
$$= \frac{27}{2}$$
$$\text{Area}_2 = \int_{-1}^{2} (x^2 - 2)\, dx$$
$$= \left[\frac{x^3}{3} - 2x\right]_{-1}^{2}$$
$$= \left[\left(\frac{(2)^3}{3} - 2(2)\right) - \left(\frac{(-1)^3}{3} - 2(-1)\right)\right]$$
$$= |-3|$$
$$\text{Area} = \frac{27}{2} + 3 = \frac{33}{2}[\text{units}^2]$$

OR

Method 3

x intercept of $g(x)$:

$x^2 - 2 = 0 \ \to x = \sqrt{2}$

$A_1 = \left|\int_{-1}^{\sqrt{2}} (x^2 - 2) dx\right| = \frac{4\sqrt{2}+5}{3}$

$A_2 = \int_{\sqrt{2}}^{2} (x^2 - 2) dx = \frac{4\sqrt{2}-4}{3}$

$A_3 = \int_{-1}^{2} (x + 4) dx = \frac{27}{2}$

Total Area $= \frac{4\sqrt{2}+5}{3} + \frac{27}{2} - \left(\frac{4\sqrt{2}-4}{3}\right) = \frac{33}{2}[\text{units}^2]$

</td><td>

Scale 15D (0, 4, 8, 12, 15)

Consider as 4 steps:

1. Integrate f
2. Integrate g
3. Combine
4. Evaluate with Limits

They may do these in a different order, e.g., combine f and g and then integrate this function (= 3 steps)

Low Partial Credit:

- Integration indicated
- Some correct integration
- Sets up integration of original function(s)
- Some work of merit towards finding an approximate area using the Trapezoidal Rule
- Work of merit towards finding x intercept of $g(x)$
- $f(-1)$ or $f(2)$ found

Mid Partial Credit:

- 2 steps correct
- 1 relevant area calculated

High Partial Credit:

- 2 relevant areas calculated correctly
- Steps 1 and 2 correct and one relevant area calculated
- 3 steps correct

Full Credit -1

- Area $= -16.5$

</td></tr>
</table>

Q6	Model Solution – 30 Marks	Marking Notes
(b)	$\left[b\left(\frac{1}{b}e^{bx}\right)\right]_0^b = e$ $\left(e^{b(b)} - e^{b(0)}\right) = e$ $e^{b^2} - 1 = e$ $e^{b^2} = e + 1$ $b^2 = \ln(e+1)$ $b = \sqrt{\ln(e+1)} = 1 \cdot 15 \ldots.$	**Scale 5D (0, 2, 3, 4, 5)** Note: if there is an error in integration, max of Mid Partial Credit is awarded and only if both limits are substituted correctly *Low Partial Credit:* • Some correct integration, for example, ke^{bx} $(k \neq 1)$ appears without the integral sign • $+C$ *Mid Partial Credit:* • Fully correct integration *High Partial Credit:* • $e^{b^2} - 1 = e$

Q7	Model Solution – 50 Marks	Marking Notes
(a)	$v(0) = \frac{2}{3}(0)^3 - 6(0)^2 + 13(0) + 109$ $v(0) = 109$ km/hr	**Scale 5B (0, 2, 5)** Accept correct answer without work. *Partial Credit:* • Some correct substitution • Identifies $t = 0$ • Substitutes $t = 5$ *Full Credit -1* • No unit or incorrect unit
(b)	$v'(t) = 2t^2 - 12t + 13$ $v'(5) = 2(5)^2 - 12(5) + 13$ $= 3$ [km/hr/min]	**Scale 10C (0, 4, 7, 10)** Accept correct answer without unit *Low Partial Credit:* • Some correct differentiation *High Partial Credit* • Substitution into correct derivative • At most one error in differentiation and finishes correctly

Q7	Model Solution – 50 Marks	Marking Notes
(c)	Maximum speed when $v'(t) = 0$ $2t^2 - 12t + 13 = 0$ $t = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(2)(13)}}{2(2)}$ $t = 4 \cdot 58$ or $t = 1 \cdot 42$ Maximum at $t = 1 \cdot 42$ [as coefficient of $t^3 > 0$ and domain of interest is $[0, 4]$, with local min at $t = 4 \cdot 58$] **OR** [$v''(t) = 4t - 12$ $v''(1.42) < 0$ $\Rightarrow$ maximum at $t = 1.42$]	**Scale 10C (0, 4, 7, 10)** Note: Accept candidate's derivative from part (b) Note: If candidate's derivative is linear, award *Low Partial Credit* at most 3 steps: **1.** $\boldsymbol{v'(t) = 0}$ **2.** Substitutes into formula **3**. Evaluates for $\boldsymbol{t}$ *Low Partial Credit:* • Work of merit in finding $v'(t)$ or brings $v'(t)$ from (b) • States $v'(t) = 0$ or similar • $v''(t)$ appears *High Partial Credit:* • 2 steps correct *Full Credit –1* • $t = 4.58$ written down and not explicitly excluded • Rounded incorrectly or no rounding, otherwise correct

Q7	Model Solution – 50 Marks	Marking Notes
(d)	$\frac{1}{5-0}\left[\int_0^5\left(\frac{2}{3}t^3-6t^2+13t+109\right)dt\right]$ $=\frac{1}{5}\left[\frac{t^4}{6}-2t^3+\frac{13t^2}{2}+109t\right]_0^5$ $=\frac{1}{5}\left[\left(\frac{(5)^4}{6}-2(5)^3+\frac{13(5)^2}{2}+109(5)\right)-(0)\right]$ $=112\cdot333$ …. km/hr $=112\cdot33$ km/hr [2 d.p.]	**Scale 10D (0, 3, 5, 8, 10)** Note: Indication of integration is required to be awarded any credit **4 steps:** Note: If $\frac{1}{5}$ is omitted, treat step 1 as not fully correct, but all other steps can be accepted as correct Note: If speed is treated as $\boldsymbol{v'(t)}$ in (a) a correct solution must include the line, $\frac{1}{5}\left[\int_0^5 \boldsymbol{v'(t)dt}\right]$ **1.** $\frac{1}{5}\left[\int_0^5 v(t)dt\right]$ **2.** Integrates correctly **3.** Subs in limits **4.** Evaluates correctly *Low Partial Credit:* • Work of merit, for example, integration indicated *Mid Partial Credit:* • 2 steps correct *High Partial Credit* • 3 steps correct *Full Credit -1* • Rounded incorrectly or no rounding, otherwise correct • Incorrect unit or no unit, otherwise correct

Q7	Model Solution – 50 Marks	Marking Notes
(e)	Answer: B Justification: $v'(1) > 0$: Function is increasing so the slope is positive. $v''(1) < 0$: Rate of increase is slowing. **OR** The slope is decreasing.	**Scale 5C (0, 2, 3, 5)** Note: Justification needs to explicitly deal with both $\boldsymbol{v'}$ and $\boldsymbol{v''}$, but can be a single combined sentence. Note: Substitution into $\boldsymbol{v'(t)}$ or $\boldsymbol{v''(t)}$ is not considered work of merit. Note: Accept "the function is concave down", or similar as a justification using $\boldsymbol{v''(1) < 0}$ 3 elements required: **1.** Answer B **2**. Justification for $\boldsymbol{v'(1) > 0}$ **3.** Justification for $\boldsymbol{v''(1) < 0}$ *Low Partial Credit:* • Answer correct • Work of merit in either justification *High Partial Credit:* • 2 elements correct • Answer given as B or D, justified correctly using $v'(1)$ • Answer given as B justified correctly using $v''(1)$ • Answer given as A justified correctly using $v''(1)$ • No answer given, but 2 justifications are correct

(f)	Time $=$ Distance/Speed $= \frac{10}{100} = 6$ [minutes]	**Scale 5B (0, 2, 5)** Note: Accept correct answer without units *Partial Credit:* • Work of merit in finding time *Full Credit –1* • $1/10$ (i.e. incorrect unit)
(g)	<u>120 km/hr for 2 minutes:</u> Distance $= 120 \times \frac{2}{60} = 4\text{km}$ $10 - 4 = 6$ km remaining to get to B <u>Average Speed:</u> $\frac{10}{\text{total time}} = 100$ $\text{total time} = \frac{1}{10}\text{hrs}$ $= 6 \text{ minutes}$ ⇒ 4 minutes remaining to get to B <u>Average speed for last 6 km:</u> $\text{Avg Speed} = 6 \div \left(\frac{1}{15}\right) = 90 \text{ km/hr}$ $\frac{120+v}{2} = 90$, where v is the speed at B $v = 60\ km/hr$ Decelerates from 120 to 60 over 4 minutes. So, deceleration $=$ 15 km/hr per minute **OR** Average speed for last 6 km: $\frac{120+v}{2}$, where v is the speed at B $\text{Distance} = \left(\frac{120+v}{2}\right) \times \text{time}$ $6 = \left(\frac{(120+v)}{2}\right) \times \frac{4}{60}$ $v = 60$ Decelerates from 120 to 60 over 4 minutes. Deceleration $=$ 15 km/hr per minute **OR** Average speed for the last 6km: $\frac{1}{4}\int_0^4 (120 - at)dt = 90$ $\frac{1}{4}\left[120t - \frac{1}{2}at^2\right]_0^4 = 90$ $\frac{1}{4}\left[120(4) - \frac{1}{2}a(4)^2\right] = 90$ $120 - 2a = 90$ $a = 15\text{kmh}^{-1}$ per minute	**Scale 5D (0, 2, 3, 4, 5)** Accept -15km/hr/min *Low Partial Credit:* • Some correct substitution in relation to distance for first 2 minutes or time / distance for remaining part • Indicates integration *Mid Partial Credit:* • Identifies 4 km, 6 km or 4 minutes. *High Partial Credit* • Calculates 60 km/hr after the extra 4 minutes or 90km/hr after extra 2 minutes. *Full Credit -1* • Answer in km/hr/hr

Q8	Model Solution – 50 Marks	Marking Notes
(a)	$F = 3000(1 + 0.024)^5$ $= €3377.70$	**Scale 5B (0, 2, 5)** *Partial Credit:* • Work of merit, for example, some correct substitution into relevant formula; finds 2.4% as a decimal
(b) **(i),** **(ii)**	**(i)** It is the amount that should be invested today to amount to €1000 in 1 years' time at the particular interest rate. **(ii)** $4000 = P(1 + 0.024)^6$ $\dfrac{4000}{1.024^6} = P$ $P = €3469 \cdot 45$	**Scale 10D (0, 3, 5, 8, 10)** Note: In (i) Accept $\boldsymbol{P} = \frac{\mathbf{1000}}{(\mathbf{1}+\boldsymbol{i})}$ *Low Partial Credit:* • Work of merit in (i) or (ii), for example, formula in (i), correct substitution into relevant formula in (ii) • 2.4% written as a decimal *Mid Partial Credit:* • (i) or (ii) correct • Work of merit in both parts *High Partial Credit* • One part correct and work of merit in the other part
(c)	$1 \cdot 024 = (1 + i)^4$ $(1.024)^{\frac{1}{4}} = 1 + i$ $(1.024)^{\frac{1}{4}} - 1 = i$ $0 \cdot 005947.. = i$ Rate $= 0 \cdot 59\%$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Some correct substitution into relevant formula • 2.4% written as a decimal *High Partial Credit:* • $(1.024)^{\frac{1}{4}} = 1 + i$ • Evaluates correctly $i = (1.024)^4 - 1$ • Uses 3 or 12 instead of 4, but otherwise correct *Full Credit -1* • Answer given as a decimal

Q8	Model Solution – 50 Marks	Marking Notes
(d) **(i)** **(ii)**	**(i)** $A(1\cdot0011)^{36}+A(1\cdot0011)^{35}+\cdots$ $\ldots+A(1\cdot0011)^{2}+A(1\cdot0011)$ **OR** $=A[(1\cdot0011)^{36}+(1\cdot0011)^{35}+\cdots$ $\ldots+(1\cdot0011)^{2}+(1\cdot0011)]$ **(ii)** $A[1\cdot0011+(1\cdot0011)^{2}+\cdots$ $\ldots+(1\cdot0011)^{35}+(1\cdot0011)^{36}]$ $a=1\cdot0011,\ \ r=1\cdot0011,\ \ \ n=36$ $A\left[\frac{1\cdot0011\,(1-1\cdot0011^{36})}{1-1\cdot0011}\right]=12000$ $A=\dfrac{12000}{\frac{1\cdot0011(1-1\cdot0011^{36})}{1-1\cdot0011}}$ $A=€326\cdot60$ [2 D.P.]	**Scale 15D (0, 4, 8, 12, 15)** Consider as requiring 3 steps: **1.** Finds geometric series **2.** Substitutes into geometric formula **3.** Finds A *Low Partial Credit:* • Work of merit in either part, for example, in **(i)**Writes 0.11% as a decimal; in **(ii)**, sets answer in **(i)** equal to 12000 *Mid Partial Credit:* • 1 step correct • Substantial work of merit in both parts *High Partial Credit:* • 2 steps correct *Full Credit –1:* • Correct solution, but excludes second and/or second last term • Investments made at the end of each month, otherwise correct
(e)	$E(x)=11(0\cdot52)+(x-5)(0\cdot15)$ $+x(0\cdot33)=13\cdot85$ $0{\cdot}15x+0{\cdot}33x=13{\cdot}85-5{\cdot}72+0{\cdot}75$ $0\cdot48x=8\cdot88$ $x=€18\cdot50$	**Scale 10C (0, 4, 7, 10)** *Low Partial Credit:* • Work of merit, for example, some correct term in $E(x)$ *High Partial Credit* • Fully correct equation
(f)	Cost Price = 82% of Selling Price Profit = 18% of Selling Price Mark-up $=\frac{0{\cdot}18}{0{\cdot}82}\times100$ $=0\cdot2195=22\%$ [nearest percent] **OR** Let $x=$ selling price and $y=$ cost price $\dfrac{x-y}{x}=0.18\ \rightarrow y=0.82x$ Mark up: $\frac{x-y}{y}=\frac{x-0.82x}{0.82x}=\frac{9}{41}$ Mark up: $\frac{9}{41}\times100=21.95$ = 22% [nearest percent]	**Scale 5B (0, 2, 5)** *Partial Credit:* • Work of merit, for example, states CP = 82% of SP • Mentions 82% • Finds 18% of a number

<table>
<tr><th>Q9</th><th>Model Solution – 50 Marks</th><th>Marking Notes</th></tr>
<tr><td>(a)
(i) (ii)
(iii)</td><td>(i) $2^0,\ 2^1,\ 2^2,\ 2^3,\ 2^4$

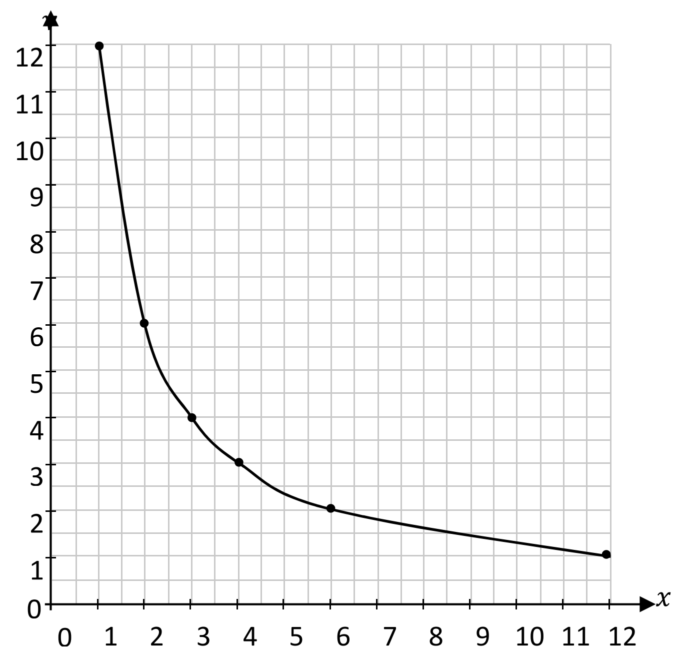
OR

$\mathbf{1},\ \mathbf{2},\ \mathbf{4},\ \mathbf{8},\ \mathbf{16}$

(ii) 8

(iii) 2^{10} and 3^{12} have no common factors
2^{10} will have 11 factors
3^{12} will have 13 factors

So $2^{10} \times 3^{12}$ will have $(11)(13) = 143$ factors</td><td>Scale 15D (0, 4, 8, 12, 15)
Accept for full credit correct answers without work
In (i) accept any 5 factors including negative factors
In (ii) accept 16 for full credit
In (iii) accept 286 factors for full credit
Low Partial Credit:
• Work of merit in one part, for example, one correct factor in (i) or lists 2 or more factors in (ii)
• Some valid relevant statement in (ii) or (iii)
Mid Partial Credit
• One part correct
High Partial Credit
• Two parts correct
• Either (i) or (ii) correct and work of merit in (iii)</td></tr>
<tr><td>(b)
(i)
(ii)
(iii)</td><td>
<table>
<tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>6</td><td>12</td></tr>
<tr><td>y</td><td>12</td><td>6</td><td>4</td><td>3</td><td>2</td><td>1</td></tr>
</table>

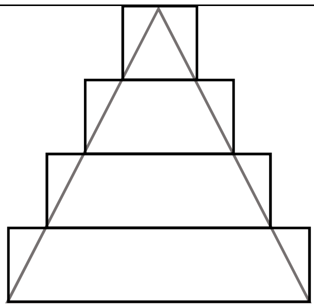


</td><td>Scale 20D (0, 5, 10, 15, 20)
Solution consists of 12 parts:
• 6 values in table
• 5 points plotted
• Points joined appropriately (not with line segments)
Low Partial Credit:
• 3 parts correct
Mid Partial Credit
• 7 parts correct
High Partial Credit
• 9 parts correct

Full Credit -1
• 11 parts correct
• Correct graph with no table entries</td></tr>
</table>

(c)(i)	**Method 1** Derivative: $y = \frac{12}{x} \quad \frac{dy}{dx} = -\frac{12}{x^2}$ $\frac{dy}{dx}(x = p) = -\frac{12}{p^2}$ Equation at point $\left(p, \frac{12}{p}\right)$: $y - \frac{12}{p} = -\frac{12}{p^2}(x - p)$ $y = -\frac{12}{p^2}x + \frac{24}{p}$ **OR** **Method 2** Derivative: $y = \frac{12}{x} \quad \frac{dy}{dx} = -\frac{12}{x^2}$ $\frac{dy}{dx}(x = p) = -\frac{12}{p^2}$ Equation at point $\left(p, \frac{12}{p}\right)$: Line is of the form $y = -\frac{12}{p^2}x + c$ $\frac{12}{p} = -\frac{12}{p^2}(p) + c$ $\left[\frac{12}{p} = -\frac{12}{p} + c\right]$ $c = \frac{24}{p}$ **OR** **Method 3** From the given equation the slope is $-\frac{12}{p^2}$ $\frac{dy}{dx} = -\frac{12}{x^2}$ $\frac{dy}{dx}(x = p) = -\frac{12}{p^2}$ Substitute $\left(p, \frac{12}{p}\right)$ in the given equation $\frac{12}{p} = -\frac{12}{p^2}(p) + \frac{24}{p}$ $\frac{12}{p} = -\frac{12}{p} + \frac{24}{p}$ $\frac{12}{p} = \frac{12}{p}$	**Scale 10D (0, 3, 5, 8, 10)** **Method 1 & Method 2** 4 steps: **1.** Finds $\boldsymbol{\frac{dy}{dx}}$ **2.** Finds slope at $\boldsymbol{x = p}$ **3.** Subs slope and point $\boldsymbol{\left(p, \frac{12}{p}\right)}$ into equation of line formula **4.** Finds equation in required form *Low Partial Credit:* • Work of merit, for example, some correct differentiation, some correct substitution into equation of line formula *Mid Partial Credit* • 2 steps correct *High Partial Credit* • 3 steps correct **Method 3** 4 steps: **1.** From the given equation writes down the slope of the tangent **2.** Finds $\boldsymbol{\frac{dy}{dx}}$ **3.** Substitutes $\boldsymbol{x = p}$ in $\boldsymbol{\frac{dy}{dx}}$ **4.** Verifies that the point $\boldsymbol{\left(p, \frac{12}{p}\right)}$ is on the given equation *Low Partial Credit:* • Work of merit, for example, some correct differentiation, some correct substitution into the given equation *Mid Partial Credit* • 2 steps correct. *High Partial Credit* • 3 steps correct.

(c)(ii)	<u>y-intercept:</u> $y = -\frac{12}{p^2}(0) + \frac{24}{p}$ $y = \frac{24}{p}$ $\left(0,\ \frac{24}{p}\right)$ $\text{height} = \frac{24}{p}$ <u>x-intercept:</u> $0 = -\frac{12}{p^2}x + \frac{24}{p}$ $\frac{12}{p^2}x = \frac{24}{p}$ $\frac{12}{p}x = 24$ $x = 2p$ $(2p,\ 0)$ $\text{base} = 2p$ Area $= \frac{1}{2}(2p)\left(\frac{24}{p}\right)$ $= 24 \text{ units}^2$ **OR** $x \text{ intercept } (2p, 0)$ $\int_0^{2p}\left(-\frac{12}{p^2}x + \frac{24}{p}\right)dx$ $= \left(-\frac{12x^2}{2p^2} + \frac{24x}{p}\right)_0^{2p}$ $= -24 + 48$ $= 24\ [\text{units}^2]$	**Scale 5D (0, 2, 3, 4, 5)** **Method 1** **4 steps:** **1.** Finds the x-intercept **2.** Finds the y-intercept **3.** Substitution into area formula **4.** Finds k Note: Accept where candidates substitute a value for *p* into the equation of the tangent, find both intercepts of the subsequent equation, and then find *k* *Low Partial Credit:* • Work of merit, for example, lets $x = 0$ or $y = 0$ *Mid Partial Credit* • 2 steps correct *High Partial Credit* • 3 steps correct **Method 2** **1.** Finds x intercept **2.** Integrates the function **3.** Substitutes limits **4.** Finds k *Low Partial Credit:* • Work of merit, for example, lets $y = 0$ • Indicates integration *Mid Partial Credit* • 2 steps correct *High Partial Credit* • 3 steps correct

Q10	Model Solution – 50 Marks	Marking Notes
(a)		**Scale 5B (0, 2, 5)** *Partial Credit:* • Work of merit, for example, each rectangle of height 2 units
(b)	**Method 1** $h = \frac{8}{3}$ Using similar triangles $\frac{w_1}{2} = \frac{8}{3} \div 8$ (w_1 = length of middle rectangle) $w_1 = \frac{2}{3}$ $w_2 = \frac{4}{3}$ (w_2 = length of top rectangle) Area $= 2\left(\frac{8}{3}\right) + \frac{4}{3}\left(\frac{8}{3}\right) + \frac{2}{3}\left(\frac{8}{3}\right)$ $= \frac{32}{3}$ units^2 **Method 2** 6 small triangles of length $\frac{2}{6} = \frac{1}{3}$ $\text{Area}_{\text{small }\Delta'\text{s}} = 6 \times \frac{1}{2} \times \frac{1}{3} \times \frac{8}{3} = \frac{8}{3}$ $\text{Area}_{\text{big }\Delta} = \frac{1}{2} \times 2 \times 8 = 8$ $\text{Area}_{\text{rectangles}} = \frac{8}{3} + 8 = \frac{32}{3}$ units^2 **Method 3** Sum of lengths of horizontal sides of small triangles (i.e., excess of rectangles over large triangle) $= 2$ Height of each small triangle $= \frac{8}{3}$ $\sum$ areas of small Δ's $= \frac{1}{2} \times 2 \times \frac{8}{3} = \frac{8}{3}$ Area of large triangle $= \frac{1}{2} \times 2 \times 8 = 8$ $\therefore$ Area of rectangles $= \frac{8}{3} + 8 = \frac{32}{3}$	**Scale 10C (0, 4, 7, 10)** *Low Partial Credit:* • Work of merit in finding dimensions **or** area of one rectangle or one small triangle, • Finds $h = \frac{8}{3}$ • Finds the area of the large triangle *High Partial Credit:* • w_1 and w_2 found • Areas of 2 rectangles found • Sum of the areas of the small triangles found with work shown

Q10	Model Solution – 50 Marks	Marking Notes
(c)	**Method 1** $T_4 = \frac{8}{4}\left[\frac{2}{4} + \frac{4}{4} + \frac{6}{4} + \frac{8}{4}\right]$ Similarly, $T_n = \frac{8}{n}\left[\frac{2}{n} + \frac{4}{n} + \cdots + \frac{2n}{n}\right]$ $= \frac{8}{n^2}[2 + 4 + 6 + \cdots + 2n]$ $2 + 4 + \cdots + 2n$ is an A.P. with $a = 2$ and $d = 2$ and n terms $S_n = \frac{n}{2}[2(2) + (n-1)(2)]$ $= n(n+1)$ $T_n = \frac{8}{n^2}[n(n+1)]$ $= \frac{8}{n}(n+1)$ or equivalent **Method 2** Total area = area of given triangle plus sum of areas of small triangles. $h = \frac{8}{n}$ $2n$ small triangles Base of each small $\Delta = \frac{2}{2n} = \frac{1}{n}$ $\text{Area of small } \Delta's = 2n \times \frac{1}{2} \times \frac{1}{n} \times \frac{8}{n}$ $= \frac{8}{n}$ Area $= \left(\frac{8}{n}\right) + \left(\frac{1}{2} \times 2 \times 8\right)$ $= \frac{8}{n} + 8$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Work of merit, for example, work towards establishing pattern by writing $T_k,\ k \neq 3$ • $\frac{8}{n}$ mentioned as height of rectangle • Identifies $2n$ (or n) small triangles • Base length of each small triangle found *High Partial Credit:* • $T_n = \frac{8}{n}\left[\frac{2}{n} + \frac{4}{n} + \cdots + \frac{2n}{n}\right]$ or equivalent not in closed form • Finds sum of the areas of the small triangles

Q10	Model Solution – 50 Marks	Marking Notes
(d)	$\frac{8(n-1)}{n} > 0 \cdot 95(8)$ $\frac{n-1}{n} > 0 \cdot 95$ $n - 1 > 0 \cdot 95n$ $0 \cdot 05n > 1$ $n > 20$ $n = 21$ **OR** $\frac{8n^2(n-1)}{n} > 0.95(8)n^2$ $8n^2 - 7.6n^2 - 8n > 0$ $0.4n^2 - 8n > 0$ $n^2 - 20n > 0$ $n(n-20) > 0$ $n > 20$ $n = 21$	**Scale 10D (0, 3, 5, 8, 10)** Note: where candidates multiply both sides by n^2, they must find $n = 20$ to be awarded *High Partial Credit.* *Low Partial Credit:* • Work of merit in establishing inequality, for example, finds the area of triangle *Mid Partial Credit* • Forms the correct inequality *High Partial Credit* • $n - 1 > 0 \cdot 95n$
(e) (i)	$\int_0^h \frac{x^2 c^2}{h^2} dx = \frac{c^2}{h^2} \int_0^h x^2 dx$ $= \frac{c^2}{h^2} \left[\frac{x^3}{3}\right]_0^h$ $= \frac{c^2}{h^2} \left[\frac{h^3}{3} - 0\right]$ $= \frac{c^2 h}{3}$	**Scale 10C (0, 4, 7, 10)** *Low Partial Credit:* • Integral set up correctly *High Partial Credit* • Integration is correct • Mishandles $\frac{c^2}{h^2}$, but otherwise correct.

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(e)(ii)	$\frac{dx}{dt} = 3 \quad \frac{dS}{dt} = \frac{dS}{dx} \times \frac{dx}{dt}$ $S(x) = \frac{x^2\, c^2}{h^2}$ $\frac{dS}{dx} = \frac{c^2}{h^2}(2x)$ $\frac{dS}{dt} = \frac{c^2}{h^2}(2x)(3)$ $= \frac{6c^2x}{h^2}$ When $x = \frac{h}{2}$ $\frac{dS}{dt} = \frac{6c^2\left(\frac{h}{2}\right)}{h^2} = \frac{3c^2}{h}$	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* • States a relevant derivative, for example, $\frac{dS}{dx}$ or $\frac{dS}{dt}$ • $x = \frac{h}{2}$ • Some correct differentiation *Mid Partial Credit* • Any **two** of the following: ○ $\frac{dx}{dt} = 3$ ○ $x = \frac{h}{2}$ ○ $\frac{dS}{dx} = \frac{c^2}{h^2}(2x)$ ○ $\frac{ds}{dt} = \frac{dS}{dx} \times \frac{dx}{dt}$ or similar *High Partial Credit* • $\frac{ds}{dt} = \frac{dS}{dx} \times \frac{dx}{dt}$, and any two others from the *MPC* list above