

3.6

## Conjugate roots

$$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

note:

eg.  $\sqrt{-9} = 3i$

2. Solve these equations, giving your answers in the form  $a \pm bi$ ,  $a, b \in R$ .

(i)  $z^2 - 2z + 17 = 0$

(ii)  $z^2 + 4z + 7 = 0$

$$\begin{aligned} a &= 1 \\ b &= -2 \\ c &= 17 \end{aligned}$$

$$z = \frac{+2 \pm \sqrt{(-2)^2 - 4(1)(17)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{-64}}{2}$$

$$= \frac{2 \pm 8i}{2} = 1 \pm 4i \quad \checkmark$$

$$\begin{aligned} a &= 1 \\ b &= 4 \\ c &= 7 \end{aligned}$$

$$z = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(7)}}{2(1)}$$

$$= \frac{-4 \pm \sqrt{-12}}{2} = \frac{-4 \pm 2\sqrt{3}i}{2}$$

$$= -2 \pm \sqrt{3}i \quad \checkmark$$

3. Form a quadratic equation, given a pair of roots in each case.

(i)  $1 \pm 3i$       (ii)  $-2 \pm i$       (iii)  $4 \pm 2i$       (iv)  $\pm 5i$

(ii)

$$R_1 + R_2 = -2 + i + (-2 - i) = -4$$

$$R_1 R_2 = (-2 + i)(-2 - i) = 4 + 1 = 5$$

$$X^2 - (R_1 + R_2)X + R_1 R_2 = 0$$

Diff. 2 Squares!

$$X^2 - (-4)X + 5 = 0$$

$$X^2 + 4X + 5 = 0 \quad \checkmark$$

(iii)

$$R_1 + R_2 = 4 + 2i + 4 - 2i = 8$$

$$R_1 R_2 = (4 + 2i)(4 - 2i) = 16 - 4i^2 = 20$$

$$X^2 - 8X + 20 = 0 \quad \checkmark$$

4. If  $z = 4 - i$  is a root of the equation  $z^2 - 8z + 17 = 0$ , show that  $\bar{z}$  is also a root.

$\bar{z}$ -conjugate

If  $k$  is Root

$$f(k) = 0$$

$$\bar{z} = 4 + i$$

$$(4+i)^2 - 8(4+i) + 17 \stackrel{?}{=} 0$$

$$16 + 8i + i^2 - 32 - 8i + 17 \stackrel{?}{=} 0$$

$$32 - 32 = 0 \quad \text{TRUE}$$

5. Show that  $-2 + 2i$  is a root of the equation  $z^3 + 3z^2 + 4z - 8 = 0$  and find the other roots.

Plan?

- ① Sub in  
 $f(k) = 0$

- ② Cubic  
 $\Rightarrow$  Roots

Conjugate  
is 2nd Root

- ③ Find  
quadratic  
factor

$$(-2+2i)^3 + 3(-2+2i)^2 + 4(-2+2i) - 8 = 0$$

$$(-2+2i)(4-8i+4i^2) + 3(0-8i) + -8 + 8i - 8$$

$$+ 16i \pm 16i + -24i - 16 + 8i$$

$$24i - 24i = 0 \quad \checkmark$$

2 Roots are  $-2+2i$  &  $-2-2i$

$$z^2 - (R_1 + R_2)z + (R_1 R_2) = 0$$

$$z^2 - (-4)z + (4 \pm 4i^2) = 0$$

$$\underbrace{z^2 + 4z + 8}_{\text{QUADRATIC FACTOR}} = 0$$

5. Show that  $-2 + 2i$  is a root of the equation  $z^3 + 3z^2 + 4z - 8 = 0$  and find the other roots.

Plan?

④ Divide by quadratic factor

⑤ Change linear factor into root

$$\begin{array}{r} z-1 \\ \hline z^2 + 4z + 8 \) z^3 + 3z^2 + 4z - 8 \\ \cancel{+ z^3 + 4z^2 + 8z} \\ -z^2 - 4z - 8 \end{array}$$

$(z-1)$  is factor

$$z=1$$

This is the third solution

HW p-116

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Q 6, 8, 11