Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate 2021

Marking Scheme

Mathematics

Higher Level

Note to teachers and students on the use of published marking schemes

Marking schemes published by the State Examinations Commission are not intended to be standalone documents. They are an essential resource for examiners who receive training in the correct interpretation and application of the scheme. This training involves, among other things, marking samples of student work and discussing the marks awarded, so as to clarify the correct application of the scheme. The work of examiners is subsequently monitored by Advising Examiners to ensure consistent and accurate application of the marking scheme. This process is overseen by the Chief Examiner, usually assisted by a Chief Advising Examiner. The Chief Examiner is the final authority regarding whether or not the marking scheme has been correctly applied to any piece of candidate work.

Marking schemes are working documents. While a draft marking scheme is prepared in advance of the examination, the scheme is not finalised until examiners have applied it to candidates' work and the feedback from all examiners has been collated and considered in light of the full range of responses of candidates, the overall level of difficulty of the examination and the need to maintain consistency in standards from year to year. This published document contains the finalised scheme, as it was applied to all candidates' work.

In the case of marking schemes that include model solutions or answers, it should be noted that these are not intended to be exhaustive. Variations and alternatives may also be acceptable. Examiners must consider all answers on their merits, and will have consulted with their Advising Examiners when in doubt.

Future Marking Schemes

Assumptions about future marking schemes on the basis of past schemes should be avoided. While the underlying assessment principles remain the same, the details of the marking of a particular type of question may change in the context of the contribution of that question to the overall examination in a given year. The Chief Examiner in any given year has the responsibility to determine how best to ensure the fair and accurate assessment of candidates' work and to ensure consistency in the standard of the assessment from year to year. Accordingly, aspects of the structure, detail and application of the marking scheme for a particular examination are subject to change from one year to the next without notice.

Contents

Paper 1: Marking Scheme

Structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	B	C	D
No of categories	3	4	5
5-mark scale	0, 2, 5	0, 2, 3, 5	
10-mark scale		0, 3, 7, 10	0, 3, 5, 8, 10
15-mark scale			0, 4, 8, 12, 15
20-mark scale			0, 5, 10, 15, 20

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

Marking scales – level descriptors

B-scales (three categories)

- response of no substantial merit (no credit)
- partially correct response (partial credit)
- correct response (full credit)

C-scales (four categories)

- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

D-scales (five categories)

- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- response about half-right (mid partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

In certain cases, typically involving incorrect rounding, omission of units, a misreading that does not oversimplify the work, or an arithmetical error that does not oversimplify the work, a mark that is one mark below the full-credit mark may also be awarded. Such cases are denoted with a * and this level of credit is referred to as *Full Credit -1*. Thus, for example, in Scale 10C, *Full Credit -1* of 9 marks may be awarded.

The only marks that may be awarded for a question are those on the scale above, or *Full Credit -1*.

A rounding penalty is applied only once in each section (a), (b), (c) etc. A penalty for an omitted unit is applied only once in each section (a), (b), (c) etc. There is no penalty for omitted units if the question specifies the unit to be used in the answer, and there is generally no penalty for an omitted euro symbol in questions involving money.

In general, accept a candidate's work in one part of a question for use in subsequent parts of the question, unless this oversimplifies the work involved.

Summary of mark allocations and scales to be applied

Section A (120)		**Section B (100)**	
Question 1 (30) (a) 10C (b) 10D (c) 10D **Question 2 (30)** (a) 10D (b) 20D **Question 3 (30)** (a) 10D (b)(i) 10C (b)(ii) 10D	**Question 4 (30)** (a) 15D (b)(i) 5B (b)(ii) 10C **Question 5 (30)** (a)(i) 10D (a)(ii) 10D (b) 10D **Question 6 (30)** (a) 10D (b) 10C (c) 10D	**Question 7 (50)** (a)(i) 5C (a)(ii) 5C (a)(iii) 5C (a)(iv) 10C (b)(i) 10C (b)(ii) 10C (b)(iii) 5C **Question 8 (50)** (a)(i) 5C (a)(ii) 15D (b)(i) 5C (b)(ii) 10C (b)(iii) 5C (c) 10D	**Question 9 (50)** (a)(i) 5C (a)(ii) 5B (a)(iii) 10C (b) 10D (c) 10D (d) 10D **Question 10 (50)** (a)(i) 5C (a)(ii) 10C (a)(iii) 5B (a)(iv) 5B (b)(i) 5B (b)(ii) 10C (b)(iii) 5C (b)(iv) 5C

Palette of annotations available to examiners

Symbol	Name	Meaning in the body of the work	Meaning when used in the right margin
✔	Tick	Work of relevance	The work presented in the body of the script merits full credit
✘	Cross	Incorrect work (distinct from an error)	The work presented in the body of the script merits 0 credit
*	Star	Rounding / Unit / Arithmetic error Misreading	
	Horizontal wavy	Error	
✔l	Tick L		The work presented in the body of the script merits low partial credit
✔m	Tick M		The work presented in the body of the script merits mid partial credit (or partial credit)
✔h	Tick H		The work presented in the body of the script merits high partial credit
F*	F star		The work presented in the body of the script merits Full Credit (– 1)
[	Left Bracket		Another version of this solution is presented elsewhere and it merits equal or higher credit
	Vertical wavy	No work on this page (portion of the page)	
O	Oversimplify	The candidate has oversimplified the work	
S	Stops early	The candidate has stopped early in this part	

Note: Where work of substance is presented in the body of the script, the annotation on the right margin should reflect a combination of annotations in the work

In a **C scale** where * and and appear in the body of the work, then ✔l should be placed in the right margin.

In the case of a **D scale** with the same annotations, ✔m should be placed in the right margin.

A ✔ in the body of the work may sometimes be used to indicate where a portion of the work presented has value and has merited one of the levels of credit described in the marking scheme. The level of credit is then indicated in the right margin.

Detailed marking notes

Model Solutions & Marking Notes

Note: The model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any Examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her Advising Examiner.

Q1	Model Solution – 30 Marks	Marking Notes
(a)	$\frac{(4-2i)}{(2+4i)} = \frac{4-2i}{2+4i} \times \frac{2-4i}{2-4i}$ $= \frac{(8-16i-4i-8)}{2^2+4^2}$ $= \frac{-20i}{20}$ $= 0-1i$ $\therefore k=-1$ **OR** $\frac{(4-2i)}{(2+4i)} = \frac{-i(2+4i)}{2+4i}$ $= 0-1i$ $\therefore k=-1$ **OR** $4-2i = ki(2+4i)$ **Re**: $4=-4k \quad \therefore k=-1$ **or Im**: $-2i = 2ki \quad \therefore k=-1$	**Scale 10C(0, 3, 7, 10)** *Note:* Accept $0-1i$ *Low Partial Credit:* • $2-4i$ • $4-2i = ki(2+4i)$ *High Partial Credit:* • $\frac{4-2i}{2+4i} \times \frac{2-4i}{2-4i}$ • $4-2i = -4k+2ki$ • Sets Re = Re **or** Im = Im *Full Credit –1:* • $0-i$ or $-1i$ as solution, with k not identified.

Q1	Model Solution – 30 Marks	Marking Notes
(b)	$-5+12i = (a+bi)^2$ $a^2+2abi-b^2$ Re: $a^2-b^2=-5$ Im: $2ab=12 \quad \therefore \quad b=\frac{6}{a}$ $a^2-\left(\frac{6}{a}\right)^2 = -5$ $a^4+5a^2-36 = 0$ $(a^2+9)(a^2-4) = 0$ $\therefore\ a=\pm 2$ and $b=\pm 3$ Answer: $2+3i,\ -2-3i$ **OR** $r=\sqrt{5^2+12^2}=13$ $\tan\theta=-\frac{12}{5}$ so $\cos\theta=-\frac{5}{13}$ $(-5+12i)^{\frac{1}{2}}$ $=[13(\cos(\theta+2n\pi)+i\sin(\theta+2n\pi))]^{\frac{1}{2}}$ $=\sqrt{13}\left(\cos\left(\frac{\theta}{2}+n\pi\right)+i\sin\left(\frac{\theta}{2}+n\pi\right)\right)$ $2\sin^2\left(\frac{\theta}{2}\right)=1-\cos\theta=1+\frac{5}{13}$ So $\sin\left(\frac{\theta}{2}\right)=\frac{3}{\sqrt{13}}$ and so $\cos\left(\frac{\theta}{2}\right)=\frac{2}{\sqrt{13}}$ $n=0$: $\sqrt{13}\left(\cos\left(\frac{\theta}{2}\right)+i\sin\left(\frac{\theta}{2}\right)\right)$ $=\sqrt{13}\left(\frac{2}{\sqrt{13}}+i\frac{3}{\sqrt{13}}\right)=2+3i$ $n=1$: $\sqrt{13}\left(\cos\left(\frac{\theta}{2}+\pi\right)+i\sin\left(\frac{\theta}{2}+\pi\right)\right)$ $=\sqrt{13}\left(-\frac{2}{\sqrt{13}}-i\frac{3}{\sqrt{13}}\right)=-2-3i$	**Scale 10D(0, 3, 5, 8, 10)** *Note*: Accept $2+3i$ for *Full Credit* *Low Partial Credit:* • $(a+bi)^2=-5+12i$ • $a+bi=(-5+12i)^{\frac{1}{2}}$ • r or θ found • $-5+12i$ plotted on Argand diagram. • Shows some knowledge of De Moivre's theorem *Mid Partial Credit:* • Relevant equation in a single variable • Writes $-5+12i$ in polar form *High Partial Credit:* • Finds $a=2$ or $b=3$ • $-2-3i$ found • Correct solution in polar form (accept with mishandling of $2n\pi$)

Q1	Model Solution – 30 Marks	Marking Notes
(c)	$z^3 = r(\cos\theta + i\sin\theta)$ $z = \left(r(\cos\theta + i\sin\theta)\right)^{\frac{1}{3}}$ $= 2\left(\cos\frac{\pi+2n\pi}{3} + i\sin\frac{\pi+2n\pi}{3}\right)$ $n = 0$: $z = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right) = 1 + \sqrt{3}i$ $n = 1$: $z = 2\left(\cos\frac{3\pi}{3} + i\sin\frac{3\pi}{3}\right) = -2$ $n = 2$: $z = 2\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right) = 1 - \sqrt{3}i$	**Scale 10D (0, 3, 5, 8, 10)** *Note*: if $((r(\cos\theta + i\sin\theta))^3$ is used, award *Low Partial Credit* at most. *Note*: polar form must be used to achieve any credit *Low Partial Credit:* • $z = \left(r(\cos\theta + i\sin\theta)\right)^{\frac{1}{3}}$ • r found • θ found • $-8 + 0i$ plotted on an Argand diagram • Shows some knowledge of De Moivre's theorem *Mid Partial Credit:* • $z = 8^{\frac{1}{3}}\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$ • $8^{\frac{1}{3}}\left(\cos\frac{\pi+2n\pi}{3} + i\sin\frac{\pi+2n\pi}{3}\right)$ *High Partial Credit:* • One root evaluated in the form $a + bi$ from De Moivre's expression • Three solutions in polar form

Q2	Model Solution – 30 Marks	Marking Notes
(a)	$\lvert -3+p \rvert = 5$ [*LPC*] $9-6p+p^2 = 25$ [*MPC*] $p^2-6p-16 = 0$ $(p+2)(p-8) = 0$ [*HPC*] $p=-2$ or $p=8$ **OR** $\lvert -3+p \rvert = 5$ $-3+p=5$ **or** $-3+p=-5$ $p=8$ **or** $p=-2$	**Scale 10D(0, 3, 5, 8, 10)** **If solving as a quadratic equation:** *Low Partial Credit:* • $x=-3$ substituted into equation *Mid Partial Credit:* • relevant quadratic in p found *High Partial Credit:* • quadratic factorised • one missing or incorrect term in quadratic, but finishes correctly. **If solving as two linear equations:** *Low Partial Credit:* • 1 linear equation • $x=-3$ substituted into equation *Mid Partial Credit:* • 1 value of p found *High Partial Credit:* • Both linear equations in p given

Q2	Model Solution – 30 Marks	Marking Notes
(b)	$f(-4) = 0$ $(-4)^3 + q(-4)^2 - 22(-4) + 56 = 0$ $-64 + 16q + 88 + 56 = 0$ $16q + 80 = 0$ $16q = -80$ $q = -5$ $x^3 - 5x^2 - 22x + 56 = 0$ $x^2(x+4) - 9x(x+4) + 14(x+4) = 0$ $(x^2 - 9x + 14)(x+4) = 0$ $(x-2)(x-7)(x+4) = 0$ Roots $= (-4,\ 2,\ 7)$ **OR** $x^2 \ - 9x \ + 14$ $x+4 \quad \sqrt{x^3 - 5x^2 - 22x + 56}$ $\underline{x^3 + 4x^2}$ $-9x^2 - 22x + 56$ $\underline{-9x^2 - 36x}$ $14x + 56$ $\underline{14x + 56}$ 0 Remainder $= 0$, $\therefore q = -5$ Roots $= (-4,\ 2,\ 7)$	**Scale 20D (0, 5, 10, 15, 20)** *Note:* If q set equal to 5 and $x+4$ divided into $f(x)$, needs conclusion in order to be accepted as showing that $q = -5$. *Low Partial Credit:* • States $f(-4) = 0$ • Any correct division • Sets up long division correctly *Mid Partial Credit:* • Shows that $q = -5$ • Correct quotient in quadratic form found (accept in terms of q) *High Partial Credit:* • Find $x = 2$ and $x = 7$ • Correct quotient in quadratic form found **and** shows that $q = -5$ Full Credit –1: • Apply a * for -4 not listed as a root

Q3	Model Solution – 30 Marks	Marking Notes
(a)	$xz = 2\sqrt{2}$ $yz = 8\sqrt{6}$ $xy = 4\sqrt{3}$ $\Rightarrow x^2y^2z^2 = (2\sqrt{2})(8\sqrt{6})(4\sqrt{3})$ $\Rightarrow x^2y^2z^2 = 384$ $\Rightarrow xyz = \sqrt{384} = 8\sqrt{6}$ [cm³] **OR** $y = \frac{4\sqrt{3}}{x}$ and $z = \frac{2\sqrt{2}}{x}$ $\Rightarrow \left(\frac{4\sqrt{3}}{x}\right)\left(\frac{2\sqrt{2}}{x}\right) = 8\sqrt{6}$ $\Rightarrow x = 1$ cm $\Rightarrow y = 4\sqrt{3}$ cm $\Rightarrow z = 2\sqrt{2}$ cm $\therefore xyz = 8\sqrt{6}$ [cm³]	**Scale 10D (0, 3, 5, 8, 10)** *Note*: $(2\sqrt{2})(8\sqrt{6})(4\sqrt{3})$ on its own is not awarded any credit. **If solving without isolating variables:** *Low Partial Credit:* • $xz = 2\sqrt{2}$, or similar • xy, xz, and yz stated • Volume $= xyz$ • $V = x(8\sqrt{6})$ or similar *Mid Partial Credit:* • $xz = 2\sqrt{2}$ **and** $yz = 8\sqrt{6}$ **and** $xy = 4\sqrt{3}$ *High Partial Credit:* • $x^2y^2z^2 = (2\sqrt{2})(8\sqrt{6})(4\sqrt{3})$ **If solving by isolating variables:** *Low Partial Credit:* • $xz = 2\sqrt{2}$, or similar *Mid Partial Credit:* • $y = \frac{4\sqrt{3}}{x}$ **and** $z = \frac{2\sqrt{2}}{x}$, or similar *High Partial Credit:* • 1 dimension found
(b) (i)	$3x^2 + 8x - 35 = 0$ $(3x - 7)(x + 5) = 0$ $x = \frac{7}{3} \quad x = -5$ **OR** Roots $= \frac{-8 \pm \sqrt{8^2 - 4(3)(-35)}}{2(3)}$ $x = \frac{7}{3} \quad x = -5$	**Scale 10C(0, 3, 7, 10)** *Low Partial Credit:* • Effort at factorisation • Quadratic formula with some substitution *High Partial Credit:* • Factors found • Formula fully substituted

Q3	Model Solution – 30 Marks	Marking Notes
(b) (ii)	$3.(3^m)^2 + 8(3^m) - 35 = 0$ [*as* $3^m > 0$*, then* $3^m \neq -5$*, so:*] $$3^m = \frac{7}{3}$$ $$\log_3 3^m = \log_3\left(\frac{7}{3}\right)$$ $$m = \log_3\left(\frac{7}{3}\right)$$ $$= \log_3 7 - \log_3 3$$ $$= \log_3 7 - 1$$	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* • Some work in writing given equation in the form of that in **(b)(i)** • −5 explicitly excluded • $x = 3^m$ *Mid Partial Credit:* • $3^m = \frac{7}{3}$ *High Partial Credit:* • m isolated correctly, for example, $m = \log_3\left(\frac{7}{3}\right)$ *Full Credit –1:* • $3^m = -5$ written down, and not explicitly excluded

Q4	Model Solution – 30 Marks	Marking Notes
(a)	$\boldsymbol{P(1)}$: $2^{3(1)-1}+3=7$, which is div. by 7 $\boldsymbol{P(k)}$: Assume $2^{3k-1}+3$ is div. by 7 $2^{3k-1}+3=7M$ $2^{3k-1}=7M-3$ $\boldsymbol{P(k+1)}$: $2^{3k+2}+3$ $=2^3(2^{3k-1})+3$ $=8(7M-3)+3$ $=56M-21$ $P(k+1)$ is divisible by 7 True for $n=1$ and, if true for $n=k$, then true for $n=k+1$. Therefore, true for all $n\geq 1$. **OR** $\boldsymbol{P(k+1)}$: $2^{3k+2}+3$ $=2^3(2^{3k-1})+3$ $=(7+1)(2^{3k-1})+3$ $=(7.2^{3k-1})+(2^{3k-1}+3)$ Both divisible by 7 True for $n=1$ and, if true for $n=k$, then true for $n=k+1$. Therefore, true for all $n\geq 1$.	**Scale 15D (0, 4, 8, 12, 15)** Accept Step $P(1)$, Step $P(k)$, Step $P(k+1)$ in any order *Low Partial Credit:* • Any **one** of Step $P(1)$, Step $P(k)$, or Step $P(k+1)$ *Mid Partial Credit:* • Any **two** of Step $P(1)$, Step $P(k)$, or Step $P(k+1)$ *High Partial Credit:* • Some valid work in using Step $P(k)$ to prove Step $P(k+1)$ *Full Credit –1:* • Omits conclusion but otherwise correct
(b) (i)	$T_n=p+(n-1)(7)$ $T_n=p+7n-7$	**Scale 5B (0, 2, 5)** *Note:* Accept $p+(n-1)(7)$. *Partial Credit:* • $a=p$ or $d=7$ • T_n formula with some substitution

Q4	Model Solution – 30 Marks	Marking Notes
(b) (ii)	$$p + 7n - 7 = 2021$$ $$p + 7n = 2028$$ $$7n = 2028 - p$$ so $2028 - p$ is the nearest multiple of 7 that is less than 2028. This is 2023. So: $$2028 - p = 2023$$ $$p = 5$$	**Scale 10C (0, 3, 7, 10)** *Low Partial Credit:* • $p + 7n - 7 = 2021$ *High Partial Credit:* • 2023 • 289

Q5	Model Solution – 30 Marks	Marking Notes
(a) (i)	$f'(x) = 6x^2 + 12x - 12$ $= 6(x^2 + 2x - 2)$ $= 6(x^2 + 2x + 1 - 1 - 2)$ $= 6[(x+1)^2 - 3]$ $= 6(x+1)^2 - 18$ $\therefore a = 6, \quad b = 1, \quad c = -18$ **OR** $6x^2 + 12x - 12 = ax^2 + 2abx + ab^2 + c$ $\Rightarrow a = 6$ $\therefore\ 2(6)(b) = 12$ $\Rightarrow b = 1$ $\therefore 6(1)^2 + c = -12$ $\Rightarrow c = -18$	**Scale 10D(0, 3, 5, 8, 10)** *Low Partial Credit:* • Any correct differentiation *Mid Partial Credit:* • Differentiation fully correct *High Partial Credit:* • any 2 of a or b or c correctly identified • $6[(x+1)^2 - 3]$ *Full Credit –1* • $6(x+1)^2 - 18$
(a) (ii)	$f'(x) > g'(x)$ $6x^2 + 12x - 12 > 36$ $6x^2 + 12x - 48 > 0$ $x^2 + 2x - 8 > 0$ $x < -4$ or $x > 2$.	**Scale 10D(0, 3, 5, 8, 10)** *Note:* Accept inclusion of -4 and 2. *Low Partial Credit:* • Any correct differentiation for $g(x)$ *Mid Partial Credit:* • Inequality correctly formulated *High Partial Credit:* • roots of quadratic found *Full Credit –1:* • $-4 > x > 2$

Q5	Model Solution – 30 Marks	Marking Notes
(b)	$h'(x) = 4\cos 2x$ $h'\left(\frac{\pi}{6}\right) = 4\cos 2\left(\frac{\pi}{6}\right) = 4\cos\left(\frac{\pi}{3}\right)$ $= 2 = m_T$ $h\left(\frac{\pi}{6}\right) = 2\sin 2\left(\frac{\pi}{6}\right) = 2\sin\left(\frac{\pi}{3}\right)$ $= \sqrt{3}$ $\left(\frac{\pi}{6}, \sqrt{3}\right) \quad m = 2$ $y - \sqrt{3} = 2\left(x - \frac{\pi}{6}\right)$ $x = 0 \Rightarrow y - \sqrt{3} = 2\left(-\frac{\pi}{6}\right)$ $y = \sqrt{3} - \frac{\pi}{3} = 0{\cdot}6848\ldots$ $\therefore k = 0{\cdot}68$ [2 D.P.] **OR** *Finds* $\left(\frac{\pi}{6}, \sqrt{3}\right)$ *and slope* $= 2$*, then:* $y = mx + c$ implies $\sqrt{3} = 2\left(\frac{\pi}{6}\right) + k$ i.e. $k = \sqrt{3} - \frac{\pi}{3}$ $= 0{\cdot}6848\ldots = 0{\cdot}68$ [2 D.P.]	**Scale 10D(0, 3, 5, 8, 10)** *Note:* Consider solution as involving 4 steps: Step 1: differentiate $2\sin 2x$ Step 2: find slope at $x = \frac{\pi}{6}$ is 2 Step 3: find y-value at $x = \frac{\pi}{6}$ Step 4: find k (for example, using slope or equation of line) *Low Partial Credit:* • Work of merit, for example, some correct differentiation; substitutes $\frac{\pi}{6}$ in to $h(x)$; formula for slope or equation of a line with some substitution *Mid Partial Credit:* • 2 steps correct *High Partial Credit:* • 3 steps correct

Q6	Model Solution – 30 Marks	Marking Notes
(a)	$h'(x) = a(x+1)(x-3)$ $= a(x^2-2x-3)$ $(0,6) \in h'(x)$ $\therefore 6 = a(0+0-3)$ $\Rightarrow a = -2$ $h'(x) = -2(x^2-2x-3)$ $h'(x) = -2x^2+4x+6$ **OR** $h'(x) = -2x^2+4x+6$ y-intercept $= h'(0) = 6$ $h'(x) = -2(x+1)(x-3)$ $\therefore$ Roots = –1 and 3 **OR** $h'(x) = ax^2+bx+c$ $h'(0) = c = 6$ So $h'(-1) = a-b+6 = 0$ and $h'(3) = 9a+3b+6 = 0$ $3 \times h'(1) = 3a-3b+18 = 0$ So $12a+24 = 0$ $\therefore a = -2$ and $b = 4$ i.e. $h'(x) = -2x^2+4x+6$	**Scale 10D (0, 3, 5, 8, 10)** *Note*: Accept 3 points from graph verified as belonging to given equation of $h'(x)$ *Note:* three points identified from graph is *Low Partial Credit*. *Low Partial Credit:* • Identifies or uses a relevant value from the graph, for example, $(x+1)$ or 3 • A factorisation of given $h'(x)$, for example $2(-x^2+2x+3)$ *Mid Partial Credit:* • Generates $(x+1)(x-3)$ from graph • Correctly verifies one point from the graph into the given $h'(x)$ • Full factorisation of given $h'(x)$ • Using simultaneous equations, finds one value (a, b, or c) *High Partial Credit:* • Generates x^2-2x-3 from graph • From given $h'(x)$, finds two roots • Correctly verifies two points from the graph into the given $h'(x)$ • From given $h'(x)$, shows that y-intercept is 6 **and** fully factorises • Using simultaneous equations, finds two values (from a, b, and c)

Q6	Model Solution – 30 Marks	Marking Notes
(b)	$h''(x) = -4x + 4 = 0$ at max/min of $h'(x)$ $\therefore x = 1$ $h'''(x) = -4 < 0$, i.e. max $h'(1) = -2(1)^2 + 4(1) + 6 = 8$ **OR** [*Quadratic with negative x^2, so max occurs halfway between the roots:*] $x = \frac{-1+3}{2} = 1$ $h'(1) = -2(1)^2 + 4(1) + 6 = 8$ **OR** $h'(x) = -2(x^2 - 2x - 3)$ $= -2(x^2 - 2x + 1 - 1 - 3)$ $= -2((x-1)^2 - 4)$ $= -2(x-1)^2 + 8$ $\therefore$ max positive slope$= 8$	**Scale 10C(0, 3, 7, 10)** *Note*: It is possible to accept for *Full Credit* without $h'''(x) < 0$ *Low Partial Credit:* • Some correct differentiation of $h'(x)$ • Finds $h''(x)$ • $h'(x) = -2(x^2 - 2x - 3)$ • Indicates axis of symmetry on graph *High Partial Credit:* • $x = 1$ • $h'(x) = -2((x-1)^2 - 4)$
(c)	$h(x) = \int h'(x)\,dx$ $h(x) = -\frac{2x^3}{3} + \frac{4x^2}{2} + 6x + C$ $(0, -2) \in h(x)$: $-2 = -0 + 0 + 0 + C$ $\Rightarrow C = -2$ $\therefore h(x) = -\frac{2x^3}{3} + 2x^2 + 6x - 2$	**Scale 10D (0, 3, 5, 8, 10)** *Note:* Accept correct answer without work. *Low Partial Credit:* • Any indication of integration *Mid Partial Credit:* • Integration of 3 terms fully correct *High Partial Credit:* • Relevant equation in C (with substitution)

<table>
<tr><th>Q7</th><th>Model Solution – 50 Marks</th><th>Marking Notes</th></tr>
<tr><td>(a)
(i)</td><td>

Swing	1	2	3	4	5
Length of Arc (cm)	45	$\mathbf{\frac{81}{2}}$	$\frac{729}{20}$	$\mathbf{\frac{6561}{200}}$	$\mathbf{\frac{59049}{2000}}$

OR

Answers as decimals:
$40{\cdot}5$, $32{\cdot}805$, and $29{\cdot}5245$</td><td>

Scale 5C(0, 2, 3, 5)

Note: Accept decimals rounded to one decimal place.

Low Partial Credit:
- 1 correct table entry

High Partial Credit:
- 2 correct table entries
</td></tr>
<tr><td>(a)
(ii)</td><td>

$$T_n = 45(0{\cdot}9)^{n-1}$$
$$T_{25} = 45(0{\cdot}9)^{24}$$
$$= 3{\cdot}5894\ldots$$
$$= 3{\cdot}6 \text{ cm} \ \ [1 \text{ D.P.}]$$
</td><td>

Scale 5C (0, 2, 3, 5)

Low Partial Credit:
- T_n formula with some substitution
- Identifies a or r

High Partial Credit:
- Formula fully substituted

Full Credit –1:
- Correct answer, no or incorrect unit
</td></tr>
<tr><td>(a)
(iii)</td><td>

$$S_{40} = \frac{45(1-0{\cdot}9^{40})}{1-0{\cdot}9}$$
$$S_{40} = 443{\cdot}3486\ldots$$
$$= 443 \text{ [cm]} \ \ [\in \mathbb{N}]$$
</td><td>

Scale 5C (0, 2, 3, 5)

Low Partial Credit:
- Sum of two or more relevant arc-lengths
- S_{40} formula with some substitution
- Identifies a or r

High Partial Credit:
- Formula fully substituted
</td></tr>
</table>

Q7	Model Solution – 50 Marks	Marking Notes
(a) (iv)	$45(0{\cdot}9)^{n-1} = 2$ $(0{\cdot}9)^{n-1} = \dfrac{2}{45}$ $n - 1 = \log_{0\cdot 9}\left(\dfrac{2}{45}\right)$ $n - 1 = 29{\cdot}5510\ldots$ $n = 30{\cdot}5510\ldots$ $\therefore p = 31$	**Scale 10C (0, 3, 7, 10)** *Note:* If solving by trial and improvement, T_{30} and T_{31} must both be evaluated for *Full Credit*. *Low Partial Credit:* • Two or more different values substituted and evaluated for n, other than 30 and 31 • $45(0{\cdot}9)^{n-1} = 2$ *High Partial Credit:* • log equation without indices • Evaluates T_{30} and T_{31} but no conclusion • $p = 31$ with T_{31} evaluated
(b) (i)	$l = 2\pi r\left(\frac{\theta}{360°}\right)$ [*degrees*] $2\pi(100)\left(\frac{\theta}{360°}\right) = 45$ $\theta = \dfrac{45 \times 360°}{200\pi} = 25{\cdot}7831\ldots°$ $= 26$ [°] [$\in \mathbb{N}$] **OR** $l = r\theta$ [*radians*] $100\theta = 45$ $\theta = 0{\cdot}45$ radians $\theta = 0{\cdot}45\left(\frac{180}{\pi}\right)° = 25{\cdot}7831\ldots°$ $= 26$ [°] [$\in \mathbb{N}$]	**Scale 10C (0, 3, 7, 10)** *Low Partial Credit:* • Formula for length of arc with some substitution • Circumference found *High Partial Credit:* • θ found in radians • Arc-length formula for θ in degrees, fully substituted • Verifies that $\theta = 26$ gives arc-length of 45 cm, to the nearest cm

Q7	Model Solution – 50 Marks	Marking Notes
(b) (ii)	$26 + 26(0{\cdot}9) + 26(0{\cdot}9)^2 + 26(0{\cdot}9)^3 + \cdots$ $S_\infty = \dfrac{26}{1 - 0{\cdot}9} = 260\ [°]$	**Scale 10C (0, 3, 7, 10)** *Note:* Accept solution based on more accurate value for θ. *Low Partial Credit:* • a or r identified • First line in model solution, or similar • S_∞ formula with some substitution *High Partial Credit:* • S_∞ formula fully substituted • S_n evaluated for a large enough value of n that gives the correct answer, when rounded to the nearest degree.
(b) (iii)	[*Half total distance and half accumulated angle occur at same point*]: Distance: $S_\infty = \frac{45}{1-0{\cdot}9} = 450$ cm Half $=$ 225 [cm] **OR** $\dfrac{26(1-0{\cdot}9^n)}{1-0{\cdot}9} = \dfrac{260°}{2}$ $1 - 0{\cdot}9^n = \dfrac{1}{2}$ $-0{\cdot}9^n = -\dfrac{1}{2}$ $n = 6{\cdot}5788 \ldots$ $\dfrac{45(1-0{\cdot}9^{6{\cdot}5788})}{1-0{\cdot}9}$ $= 224{\cdot}9996 \ldots$ $= 225$ [cm] [$\in \mathbb{N}$] **OR** Half accumulated angle $= \frac{260°}{2} = 130°$ So distance $= 2\pi(100)\left(\frac{130°}{360°}\right)$ $= 226{\cdot}89 \ldots = 227$ [cm] [$\in \mathbb{N}$]	**Scale 5C (0, 2, 3, 5)** *Note*: Accept $n = 6{\cdot}5788$ … rounded to $n = 7$ and finished ($234{\cdot}766 \ldots = 235$ [cm] [$\in \mathbb{N}$]) *Low Partial Credit:* • Finds half accumulated angle • S_n formula with some substitution • S_∞ distance formula with some substitution *High Partial Credit:* • n found • S_∞ found for distance

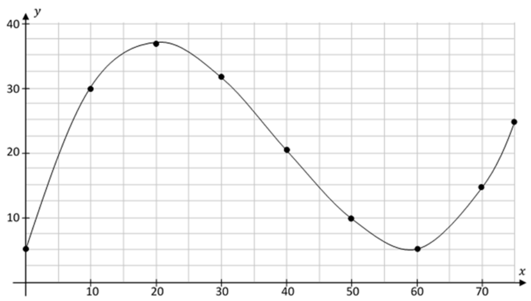
<table>
<tr><th>Q8</th><th>Model Solution – 50 Marks</th><th>Marking Notes</th></tr>
<tr><td>(a)
(i)</td><td>$h(10)$
$= 0{\cdot}001(10)^3 - 0{\cdot}12(10)^2 + p(10) + 5$
$= 30$

$10p = 36$
$p = 3{\cdot}6$</td><td>Scale 5C(0, 2, 3, 5)

Low Partial Credit:
• $h(10)$ with some relevant substitution

High Partial Credit:
• Equation in p</td></tr>
<tr><td>(a)
(ii)</td><td>
<table>
<tr><td>x</td><td>0</td><td>10</td><td>20</td><td>30</td><td>40</td></tr>
<tr><td>$h(x)$</td><td>5</td><td>30</td><td>37</td><td>32</td><td>21</td></tr>
</table>

<table>
<tr><td>x</td><td>50</td><td>60</td><td>70</td><td>75</td></tr>
<tr><td>$h(x)$</td><td>10</td><td>5</td><td>12</td><td>21·875</td></tr>
</table>



</td><td>Scale 15D (0, 4, 8, 12, 15)
14 items are required: 5 table entries and 9 plots (which also need to be joined appropriately for Full Credit)

Low Partial Credit:
• Any one item correct

Mid Partial Credit:
• any 7 items correct

High Partial Credit:
• any 11 items correct

Full Credit –1:
• All items correct but points not joined or joined incorrectly
• All items but one correct, and points appropriately joined</td></tr>
<tr><td>(b)
(i)</td><td>$h'(x) = 0{\cdot}003x^2 - 0{\cdot}24x + 3{\cdot}6$</td><td>Scale 5C (0, 2, 3, 5)

Low Partial Credit:
• correct differentiation of 1 term

High Partial Credit:
• correct differentiation of 2 terms</td></tr>
</table>

Q8	Model Solution – 50 Marks	Marking Notes
(b) (ii)	$h'(x) = 0{\cdot}003x^2 - 0{\cdot}24x + 3{\cdot}6$ $h'(20) = 0{\cdot}003(20)^2 - 0{\cdot}24(20) + 3{\cdot}6$ $= 0$, so [local] max/min at $x = 20$ $h''(20) = 0{\cdot}006(20) - 0{\cdot}24 < 0$, so local max Also $h(20) > h(0)$ and $h(20) > h(75)$ **OR** $h'(x) = 0{\cdot}003x^2 - 0{\cdot}24x + 3{\cdot}6$ $h'(20) = 0{\cdot}003(20)^2 - 0{\cdot}24(20) + 3{\cdot}6$ $= 0$, so [local] max/min at $x = 20$ From graph, turning point at $x = 20$ is a [local] max, and it is above the two endpoints [0 and 75]	**Scale 10C (0, 3, 7, 10)** *Low Partial Credit:* • $0{\cdot}003x^2 - 0{\cdot}24x + 3{\cdot}6$ • States $h'(x) = 0$, or similar *High Partial Credit:* • Shows that $h'(20) = 0$, but no further justification that it is the max in the range $[0, 75]$.
(b) (iii)	$h''(x) = 0{\cdot}006x - 0{\cdot}24 = 0$ $0{\cdot}006x = 0{\cdot}24$ $x = 40$ $h(40) = 0{\cdot}001(40)^3 - 0{\cdot}12(40)^2$ $+ 3{\cdot}6(40) + 5$ $= 21$ [m]	**Scale 5C (0, 2, 3, 5)** *Note:* work presented in (b)(iii) must involve calculus, or be based on calculus from (b)(ii), to be awarded any credit. *Low Partial Credit:* • Some correct differentiation of $h'(x)$ • $h''(x)$ indicated *High Partial Credit:* • $x = 40$

Q8	Model Solution – 50 Marks	Marking Notes
(c)	$\frac{1}{75}\int_0^{75} h(x)\ \mathrm{d}x = \frac{1}{75}\left[\frac{0{\cdot}001x^4}{4} - \frac{0{\cdot}12x^3}{3} + \frac{3{\cdot}6x^2}{2} + 5x + C\right]$ $= \frac{1}{75}\left[\frac{0{\cdot}001(75)^4}{4} - \frac{0{\cdot}12(75)^3}{3} + \frac{3{\cdot}6(75)^2}{2} + 5(75) + C - (0 + C)\right]$ $= \frac{1}{75}(1535{\cdot}15625)$ $= 20{\cdot}46875$ m $= 20{\cdot}47$ [m] [2 D.P.]	**Scale 10D (0, 3, 5, 8, 10)** *Note:* Integration is required in order to be awarded any credit *Low Partial Credit:* • Integration indicated *Mid Partial Credit:* • Integration of terms fully correct (accept without C) *High Partial Credit:* • limits substituted correctly (and $\frac{1}{75}$ present)

Q9	Model Solution – 50 Marks	Marking Notes
(a) (i)	$95 = Ae^{(-0\cdot081)(0)} + 20$ $75 = Ae^0$ $75 = A$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Some substitution into function, including $A = 75$ *High Partial Credit:* • Equation in A • Substitutes $A = 75$ and $t = 0$
(a) (ii)	*Any valid description, for example*: "The lowest temperature the coffee will cool to" **OR** "The lower limit of the coffee's temperature" **OR** "Room temperature" *etc.*	**Scale 5B (0, 2, 5)** *Partial Credit:* • reference to temperature **or** limit
(a) (iii)	$T(10) = 75e^{(-0\cdot081)(10)} + 20$ $T(10) = 53\cdot364$ ∴ Decrease $95 - 53\cdot364 = 41\cdot636$ $= 42°$ C	**Scale 10C (0, 3, 7, 10)** *Note:* Award *High Partial Credit* for finding $T'(10)$, i.e. the rate of decrease at $t = 10$. *Low Partial Credit:* • Some substitution into function • Finds $T'(t)$ *High Partial Credit:* • $T(10)$ evaluated • $T'(10)$ evaluated ($-2\cdot7025$ ...) *Full Credit –1:* • Correct answer with no or incorrect unit

Q9	Model Solution – 50 Marks	Marking Notes
(b)	$82 = 75e^{(-0\cdot081)(t)} + 20$ $62 = 75e^{-0.\cdot081t}$ $\frac{62}{75} = e^{-0\cdot081t}$ $\ln\left(\frac{62}{75}\right) = -0\cdot081t$ $t = 2\cdot3500 \ldots$ mins $t = 2$ mins 21 secs [nearest sec]	**Scale 10D (0, 3, 5, 8, 10)** *Note*: Accept $2\cdot35$ mins or 141 seconds *Low Partial Credit:* • Some substitution into equation *Mid Partial Credit* • Fully substituted equation *High Partial Credit:* • Equation in t with no indices (logs handled correctly)
(c)	$T(t) = 75e^{-0.081t} + 20$ $T'(t) = -6\cdot075e^{-0\cdot081t}$ $-6\cdot075e^{-0\cdot081t} = -4\cdot05$ $e^{-0\cdot081t} = \frac{2}{3}$ $-0\cdot081t = \ln\left(\frac{2}{3}\right)$ $t = 5\cdot0057 \ldots$ $T(5 \cdot 0057 \ldots) = 75e^{-0\cdot081(5\cdot0057\ldots)} + 20$ $= 70$ [°C]	**Scale 10D (0, 3, 5, 8, 10)** *Note:* differentiation must be used in order to be awarded any credit. *Note*: Chain rule must be applied in order to be awarded more than *Low Partial Credit* *Low Partial Credit:* • Some correct differentiation *Mid Partial Credit:* • Equation in t *High Partial Credit:* • $t = 5\cdot0057 \ldots$ or $t = 5$ • One error in finding t and finishes correctly

Q9	Model Solution – 50 Marks	Marking Notes
(d)	$\frac{dV}{dt} = -\frac{1}{20}$ $V = \frac{1}{64} = x^3$ so $x = \frac{1}{4}$ and $\frac{dV}{dx} = 3x^2$ $\frac{dx}{dt} = \frac{dV}{dt} \div \frac{dV}{dx}$ $= -\frac{1}{20} \div 3x^2 = -\frac{1}{60x^2}$ At $x = \frac{1}{4}$, $\frac{dx}{dt} = -\frac{1}{60(0{\cdot}25)^2}$ $= -\frac{4}{15}$ cm/sec	**Scale 10D (0, 3, 5, 8, 10)** *Note*: Accept if the rates of change are treated as positive. *Low Partial Credit:* • States a relevant derivative, for example, $\frac{dV}{dt}$ • $x^3 = \frac{1}{64}$ *Mid Partial Credit:* • Any **two** of the following: ○ $\frac{dV}{dt} = -\frac{1}{20}$ ○ $\frac{dV}{dx} = 3x^2$ ○ $x = \frac{1}{4}$ ○ $\frac{dx}{dt} = \frac{dV}{dt} \div \frac{dV}{dx}$, or similar *High Partial Credit:* • $\frac{dx}{dt} = \frac{dV}{dt} \div \frac{dV}{dx}$, **and** any **two** others from the *MPC* list above *Full Credit –1:* • Unit incorrect or omitted

Q10	Model Solution – 50 Marks	Marking Notes
(a) (i)	$V(t) = 60 + 41t - 3t^2 = 0$ $(-t + 15)(3t + 4) = 0$ $t = 15$ days **OR** $t = \frac{-41 \pm \sqrt{41^2 - 4(-3)(60)}}{2(-3)}$ $= \frac{-41 \pm \sqrt{2401}}{-6}$ $= 15$, as $t > 0$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • $V(t) = 0$ • Quadratic formula with some substitution *High Partial Credit:* • $V(t)$ fully factorised • Formula fully substituted *Full Credit –1:* • Gives 15 and $-\frac{4}{3}$
(a) (ii)	$V'(t) = 41 - 6t$ $V'(5) = 41 - 6(5) = 11$ litres / day	**Scale 10C (0, 3, 7, 10)** *Note:* differentiation must be used in order to be awarded any credit. *Low Partial Credit:* • Any relevant differentiation *High Partial Credit:* • $V'(5)$ substituted *Full Credit –1* • Correct answer but no or incorrect unit
(a) (iii)	$V'(t) = 0$ $41 - 6t = 0$ $41 = 6t$ $t = \frac{41}{6}$ or $6{\cdot}8333\ldots$ [$V''(t) = -6 < 0$, so max]	**Scale 5B (0, 2, 5)** *Partial Credit:* • $V'(t) = 0$ • $41 - 6t$ • $V''(t) = -6$
(a) (iv)	$V\left(\frac{41}{6}\right) = 60 + 41\left(\frac{41}{6}\right) - 3\left(\frac{41}{6}\right)^2$ $= 200{\cdot}0833\ldots$ $= 200$ [litres] [nearest litre]	**Scale 5B (0, 2, 5)** *Partial Credit:* • $V'(t) = 0$ • $41 - 6t$ • $V''(t) = -6$ • Some relevant substitution into $V(t)$

Q10	Model Solution – 50 Marks	Marking Notes
(b) (i)	$I(t) = 1{\cdot}5 + \sin\frac{\pi t}{5}$ $\sin A \geq -1$ So $I(t) \geq 0{\cdot}5$ Radius increases every year, as $I(t) > 0$	**Scale 5B (0, 2, 5)** *Partial Credit:* • Some relevant work, for example, substitutes value of $t > 0$ into $I(t)$; states $I(t) > 0$; states $\sin\frac{\pi t}{5} > -1$
(b) (ii)	**Show**: $I(6) = 1{\cdot}5 + \sin\frac{6\pi}{5} = 0{\cdot}9122\ldots$ $I(5) = 1{\cdot}5 + \sin\frac{5\pi}{5} = 1{\cdot}5$ $I(6) < I(5)$ **Explanation**: It grew less in year 6 than in year 5, *or similar*	**Scale 10C (0, 3, 7, 10)** Note: Accept without the conclusion (i.e. that $I(6) < I(5)$), as long as $I(5)$ and $I(6)$ evaluated, and valid explanation given. *Low Partial Credit:* • $I(5)$ or $I(6)$ with some substitution • Valid explanation *High Partial Credit:* • $I(5)$ and $I(6)$ with full substitution • Valid explanation **and** some substitution into $I(5)$ or $I(6)$
(b) (iii)	$r(2) = r(1) + I(2)$ $r(2) = r(0) + I(1) + I(2)$ $r(2) = 10 + 1{\cdot}5 + \sin\frac{\pi}{5} + 1{\cdot}5 + \sin\frac{2\pi}{5}$ $r(2) = 13 + \sin\frac{\pi}{5} + \sin\frac{2\pi}{5}$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • $r(2) = r(1) + I(2)$ • $r(3) = r(2) + I(3)$ • Some relevant substitution into $r(1)$ *High Partial Credit:* • $r(2) = r(0) + I(1) + I(2)$ • $r(1) = 10 + 1 \cdot 5 + \sin\frac{\pi}{5}$ *Full Credit –1:* • $r(2) = 10 + 1{\cdot}5 + \sin\frac{\pi}{5} + 1{\cdot}5 + \sin\frac{2\pi}{5}$

Q10	Model Solution – 50 Marks	Marking Notes
(b) (iv)	$r(10) = 10 + 10(1{\cdot}5) + \sin\frac{\pi}{5} + \sin\frac{2\pi}{5} + \cdots + \sin\frac{10\pi}{5}$ $= 10 + 15 + 0$ $= 25$ cm $V_2 = kV_1$ $\pi 25^2 h = k\pi 10^2 h$ $625 = 100k$ $k = 6{\cdot}25$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • $r(10)$ with some substitution • Formula for volume of a cylinder with some substitution *High Partial Credit:* • $r(10)$ fully substituted