Paper 2: Marking Scheme

Structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	B	C	D
No of categories	3	4	5
5-mark scale	0, 2, 5	0, 2, 3, 5	
10-mark scale		0, 3, 7, 10	0, 3, 5, 8, 10
15-mark scale			0, 4, 8, 12, 15
20-mark scale			0, 5, 10, 15, 20

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

Marking scales – level descriptors

B-scales (three categories)

- response of no substantial merit (no credit)
- partially correct response (partial credit)
- correct response (full credit)

C-scales (four categories)

- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

D-scales (five categories)

- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- response about half-right (mid partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

In certain cases, typically involving incorrect rounding, omission of units, a misreading that does not oversimplify the work, or an arithmetical error that does not oversimplify the work, a mark that is one mark below the full-credit mark may also be awarded. Such cases are denoted with a * and this level of credit is referred to as *Full Credit -1*. Thus, for example, in Scale 10C, *Full Credit -1* of 9 marks may be awarded.

The only marks that may be awarded for a question are those on the scale above, or *Full Credit -1*.

A rounding penalty is applied only once in each section (a), (b), (c) etc. It is explicitly indicated in the scheme where penalties for incorrect or omitted units are to be applied. There is no penalty for omitted units if the question specifies the unit to be used in the answer, and there is generally no penalty for an omitted euro symbol in questions involving money.

In general, accept a candidate's work in one part of a question for use in subsequent parts of the question, unless this oversimplifies the work involved.

Summary of mark allocations and scales to be applied

Section A (120)		**Section B (100)**	
Question 1 (30)	**Question 4 (30)**	**Question 7 (50)**	**Question 9 (50)**
(a) 15D	(a)(i) 10C	(a) 5C	(a)(i) 10D
(b) 10C	(a)(ii) 10D	(b) 15D	(a)(ii) 10D
(c) 5C	(b) 10D	(c) 10D	(b)(i) 5C
		(d) 5C	(b)(ii) 10C
Question 2 (30)	**Question 5 (30)**	(e) 10C	(b)(iii) 5C
(a) 10C	(a)(i) 15D	(f) 5C	(b)(iv) 5C
(b) 10D	(a)(ii) 5B		(b)(v) 5C
(c)(i) 5C	(b) 10C	**Question 8 (50)**	
(c)(ii) 5C		(a)(i) 10D	**Question 10 (50)**
	Question 6 (30)	(a)(ii) 10D	(a)(i) 10D
Question 3 (30)	(a) 20D	(b)(i) 5C	(a)(ii) 10C
(a) 15D	(b) 10D	(b)(ii) 10D	(a)(iii) 10D
(b)(i) 10D		(c)(i) 10D	(b) 10C
(b)(ii) 5B		(c)(ii) 5C	(c) 10D

Palette of annotations available to examiners

Symbol	Name	Meaning in the body of the work	Meaning when used in the right margin
✔	Tick	Work of relevance	The work presented in the body of the script merits full credit
✘	Cross	Incorrect work (distinct from an error)	The work presented in the body of the script merits 0 credit
*	Star	Rounding / Unit / Arithmetic error Misreading	
〰	Horizontal wavy	Error	
✔l	Tick L		The work presented in the body of the script merits low partial credit
✔m	Tick M		The work presented in the body of the script merits mid partial credit (or partial credit)
✔h	Tick H		The work presented in the body of the script merits high partial credit
F*	F star		The work presented in the body of the script merits Full Credit (– 1)
[	Left Bracket		Another version of this solution is presented elsewhere and it merits equal or higher credit
⦚	Vertical wavy	No work on this page (portion of the page)	
o	Oversimplify	The candidate has oversimplified the work	
S	Stops early	The candidate has stopped early in this part	

Note: Where work of substance is presented in the body of the script, the annotation on the right margin should reflect a combination of annotations in the work

In a **C scale** where * and 〰 and 〰 appear in the body of the work, then ✔l should be placed in the right margin.

In the case of a **D scale** with the same annotations, ✔m should be placed in the right margin.

A ✔ in the body of the work may sometimes be used to indicate where a portion of the work presented has value and has merited one of the levels of credit described in the marking scheme. The level of credit is them indicated in the right margin.

Detailed marking notes

Model Solutions & Marking Notes

Note: The model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any Examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her Advising Examiner.

Q1	Model Solution – 30 Marks	Marking Notes
(a)	$11 \times 0{\cdot}15^1 \times 0{\cdot}85^{10} = 0 \cdot 3248 \ldots$ $= 0 \cdot 325$ [3 D.P.] **OR** $11 \times \frac{3}{20} \times \left(\frac{17}{20}\right)^{10} = 0{\cdot}3248 \ldots$ $= 0 \cdot 325$ [3 D.P.]	**Scale 15D (0, 4, 8, 12, 15)** *Note:* multiplication between relevant terms is necessary to be awarded above *Low Partial Credit* *Low Partial Credit:* • $0{\cdot}15$ or $0{\cdot}85$ or $\frac{3}{20}$ or $\frac{17}{20}$ or 11 • x^{10} where $0 < x < 1$ *Mid Partial Credit:* • Three terms multiplied, two correct, from 11, $(0{\cdot}15^1)$, and $(0{\cdot}85^{10})$ • $(0{\cdot}15^1) \times (0{\cdot}85^{10})$ *High Partial Credit:* • $11 \times 0{\cdot}15^1 \times 0{\cdot}85^{10}$ or equivalent • $(0{\cdot}15^1) \times (0{\cdot}85^{10})$ evaluated $[0 \cdot 0295 \ldots]$
(b)	$P(0 \text{ or } 1 \text{ or } 2 \text{ left-footed})$ $= 0{\cdot}85^{11} + \binom{11}{1} \times 0{\cdot}15^1 \times 0{\cdot}85^{10}$ $+ \binom{11}{2} \times 0{\cdot}15^2 \times 0{\cdot}85^9$ $= 0{\cdot}1673 \ldots + 0{\cdot}3248 \ldots + 0{\cdot}2866 \ldots$ $= 0{\cdot}7787 \ldots$ $= 0{\cdot}78$ [2 D.P.]	**Scale 10C (0, 3, 7, 10)** *Low Partial Credit:* • First line of solution • $0{\cdot}15$ or $0{\cdot}85$ or $\frac{3}{20}$ or $\frac{17}{20}$ or $\binom{11}{1}$ *High Partial Credit:* • Two out of $P(0)$, $P(1)$, $P(2)$ fully substituted

Q1	Model Solution – 30 Marks	Marking Notes
(c)	From 10, $P(0 \text{ or } 1 \text{ or } 2 \text{ left-footed})$ $= 0{\cdot}85^{10} + \binom{10}{9} \times 0{\cdot}15^{1} 0{\cdot}85^{9}$ $+ \binom{10}{8} \times 0{\cdot}15^{2} 0{\cdot}85^{8}$ $= 0{\cdot}1968 \ldots + 0{\cdot}3474 \ldots + 0{\cdot}2758 \ldots$ $= 0{\cdot}82019 \ldots$ $= 0{\cdot}82$ [2 D.P.]	**Scale 5C (0, 2, 3, 5)** *Note:* Accept $82{\cdot}02\%$ *Low Partial Credit:* • First line of solution • $0{\cdot}15$ or $0{\cdot}85$ or $\frac{3}{20}$ or $\frac{17}{20}$ or $\binom{10}{1}$ *High Partial Credit:* • Two out of $P(0)$, $P(1)$, $P(2)$ fully substituted

<table>
<tr><th>Q2</th><th>Model Solution – 30 Marks</th><th>Marking Notes</th></tr>
<tr><td>(a)</td><td>$3k - 6\left(\frac{2k+2}{3}\right) + 2 = 0$
$\Rightarrow 3k - 4k - 4 + 2 = 0$
$\Rightarrow k = -2$</td><td>Scale 10C (0, 3, 7, 10)

Low Partial Credit:
• Some substitution into equation of line
High Partial Credit:
• Equation of line fully substituted</td></tr>
<tr><td>(b)</td><td>$s - 2t - 8 = 0$ so $s = 2t + 8$
$\frac{|4s+3t+6|}{\sqrt{4^2+3^2}} = 1$
$\frac{|8t+32+3t+6|}{5} = 1$
$|11t + 38| = 5$
$11t + 38 = 5$ or $11t + 38 = -5$
$\therefore\ t = -3$ or $\therefore\ t = -\frac{43}{11}$
$\therefore\ s = 2$ or $\therefore\ s = \frac{2}{11}$
OR
$\frac{|4s+3t+6|}{\sqrt{4^2+3^2}} = 1$
$|4s + 3t + 6| = 5$
$4s + 3t = -1$ or $4s + 3t = -11$
Intersection of either with $s - 2t = 8$:
$s = 2,\ t = -3$ or $s = \frac{2}{11},\ t = -\frac{43}{11}$</td><td>Scale 10D (0, 3, 5, 8, 10)
Note: Only one pair of s and t required.

Low Partial Credit:
• Some substitution of (s, t) into equation of line
• Some substitution into distance of point to line formula
• Finds one co-ordinate of point of intersection of two given lines
• Correct answer without work

Mid Partial Credit:
• Full substitution of s and t into both (1) the equation of the line and (2) the equation of the distance of a point to a line formula
• $4s + 3t = -1$ or $4s + 3t = -11$

High Partial Credit:
• $\frac{|8t+32+3t+6|}{5} = 1$, or similar
• ($4s + 3t = -1$ or $4s + 3t = -11$) and $s - 2t = 8$</td></tr>
</table>

Q2	Model Solution – 30 Marks	Marking Notes
(c) (i)	$\|AC\| = \sqrt{12^2 + 9^2} = 15$ $\|AD\| = \frac{2}{3}(15) = 10$ **OR** $\left(\frac{2 \times 16 + 1 \times 4}{2+1}, \ \frac{2 \times 11 + 1 \times 2}{2+1}\right)$ $= D(12, 8)$ $\|AD\| = \sqrt{(12-8)^2 + (8-2)^2} = 10$ **OR** Let D be (x, y). Then $\left(\frac{3x - 1 \times 4}{3-1}, \ \frac{3y - 1 \times 2}{3-1}\right) = (16, 11)$ So $D = (x, \ y) = (12, 8)$ $\|AD\| = \sqrt{8^2 + 6^2} = 10$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Some substitution into formula for $\|AC\|$ • Some substitution into formula for D • Identifies relevant translation *High Partial Credit:* • $\|AC\| = 15$ found • D found **and** $\|AD\|$ fully substituted
(c) (ii)	$\|AB\| = 33 \ \Rightarrow B \ is \ (37, 2)$. The translation $\overrightarrow{CB}$: x increases by 21 $x_E = 16 + \frac{1}{3}(21) = 23$ The translation $\overrightarrow{CB}$: y decreases by 9 $y_E = 11 - \frac{1}{3}(9) = 8$ So $E = (23, 8)$. **OR** $\left(\frac{2 \times 16 + 1 \times 37}{2+1}, \frac{2 \times 11 + 1 \times 2}{2+1}\right)$ $= E(23, 8)$ **OR** DE and AB parallel $\therefore$ equation DE: $y = 8$ Then x coordinate by translation or ratio or equation $BC \cap DE \ = 23$	**Scale 5C (0, 2, 3, 5)** *Note:* finding slant distances alone is not given credit *Low Partial Credit:* • One co-ordinate of *B* **or** *E* found • $\|DE\| = 11$ *High Partial Credit:* • B found **and** some work towards finding E • E found

Q3	Model Solution – 30 Marks	Marking Notes
(a)	$\lvert DB\rvert = \frac{1}{2}(4\sqrt{3}) = 2\sqrt{3}$ $\lvert CD\rvert = \sqrt{4^2+2^2} = \sqrt{20}$ or $2\sqrt{5}$ Radius $= \lvert CB\rvert$ $= \sqrt{(2\sqrt{5})^2 + (2\sqrt{3})^2} = 4\sqrt{2}$	**Scale 15D (0, 4, 8, 12, 15)** *Low Partial Credit:* • $\lvert DB\rvert$ (or $\lvert DA\rvert$) found • Pythagoras' Theorem with some relevant substitution • Right angle indicated at D; $[AC]$ or $[BC]$ drawn, and length r indicated *Mid Partial Credit:* • $\lvert CD\rvert$ found • $\lvert DB\rvert$ found **and** Pythagoras' Theorem with some relevant substitution *High Partial Credit:* • $\lvert DB\rvert$ **and** $\lvert CD\rvert$ found
(b) **(i)**	$x^2+y^2+4x-2y-95=0$: Centre $(-2, 1)$. Radius $= 10$ $(x-7)^2+(y-13)^2 = 25$: Centre $(7, 13)$. Radius $= 5$ $r_1 + r_2 = 15$ Distance between the two centres: $\sqrt{12^2+9^2} = \sqrt{225} = 15$ Touch Externally since $r_1 + r_2 =$ distance between two centres.	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* • Either centre **or** either radius found *Mid Partial Credit:* • Finds two from the following four: two centres and two radii *High Partial Credit:* • Both centres **and** both radii found *Full Credit –1:* • Both centres **and** both radii **and** distance between the centres found but no or incorrect conclusion

Q3	Model Solution – 30 Marks	Marking Notes
(b) (ii)	*Accept any point on the line* $y = \frac{4x+11}{3}$ *for which* $x > 4$. Slope of the two centres: $\frac{13-1}{7+2} = \frac{4}{3}$ For example: $\left(7+1,\ 13+\frac{4}{3}\right) = \left(8,\ 14\frac{1}{3}\right)$ **OR** $(-2, 1) \rightarrow (7, 13)$ given by x: $+9$, y: $+12$ So $(7, 13) \rightarrow (7+9, 13+12) = (16, 25)$ **OR** Any circle that touches c at that point must be on l (i.e. line through the two centres). Slope of the two centres: $\frac{13-1}{7+2} = \frac{4}{3}$ $y - 1 = \frac{4}{3}(x+2)$ l: $4x - 3y + 11 = 0$ now sub $x = 10$ $\Rightarrow\ \ y = 17$ One centre $= (10, 17)$.	**Scale 5B (0, 2, 5)** *Note:* no credit is awarded for just identifying either/both centres. *Partial Credit:* • Work of merit, for example: identifies that the point must be on the line containing the two centres; clearly recognises that $x > 4$ or $y > 9$; finds the point $(4, 9)$, or either ordinate; finds slope of line containing two centres *Full Credit –1:* • Answer as another point on the line joining the centres, but with $x < 4$ (i.e. touches internally)

Q4	Model Solution – 30 Marks	Marking Notes
(a) (i)	$\cos(A+B)=\cos A\,.\cos B-\sin A\,.\sin B$ $\cos(A+A)=\cos A\,.\cos A-\sin A\,.\sin A$ $\cos(2A)=\cos^2 A-\sin^2 A$	**Scale 10C (0, 3, 7, 10)** *Low Partial Credit:* • $\cos(A+B)$ formula with some substitution • Tested with one or more values of A *High Partial Credit:* • $\cos(A+A)$ $=\cos A\,.\cos A-\sin A\,.\sin A$
(a) (ii)	$\cos 2A=\cos^2 A-\sin^2 A$ $\Rightarrow \cos\theta=\cos^2\frac{\theta}{2}-\sin^2\frac{\theta}{2}$ using a right-angled triangle $\cos\frac{\theta}{2}=\frac{2}{\sqrt{5}}$ $\cos\theta=\left(\frac{2}{\sqrt{5}}\right)^2-\left(\frac{1}{\sqrt{5}}\right)^2=\frac{3}{5}$ **OR** $\cos\theta=\cos^2\frac{\theta}{2}-\sin^2\frac{\theta}{2}$ $=\left(1-\sin^2\frac{\theta}{2}\right)-\sin^2\frac{\theta}{2}$ $=1-2\sin^2\frac{\theta}{2}$ $=1-2\left(\frac{1}{\sqrt{5}}\right)^2=\frac{3}{5}$	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* • $\cos\theta=\cos^2\frac{\theta}{2}-\sin^2\frac{\theta}{2}$ • Valid work to find $\cos\frac{\theta}{2}$ from $\sin\frac{\theta}{2}$ • Some relevant substitution *Mid Partial Credit* • Finds $\cos\frac{\theta}{2}=\frac{2}{\sqrt{5}}$. Also accept $\cos\frac{\theta}{2}=\cos\left(\sin^{-1}\frac{1}{\sqrt{5}}\right)=0{\cdot}8944\ldots$ • $\cos\theta=\left(1-\sin^2\frac{\theta}{2}\right)-\sin^2\frac{\theta}{2}$ *High Partial Credit:* • $\cos\theta$ formula fully substituted

Q4	Model Solution – 30 Marks	Marking Notes
(b)	$\tan(\text{angle}) = -\sqrt{3}$, so reference angle $= 60°$ $\mathbf{150° \leq B + 150° \leq 510°}$ In Quad's 2 or 4, so angles are $\mathbf{300°}$ or $\mathbf{480°}$ $B + 150 = 300$ **or** $B + 150 = 480$ So $B = 150°$ **or** $B = 330°$ **OR** $\tan(B + 150) = \frac{\tan B + \tan 150}{1 - \tan B \tan 150}$ $= \frac{\tan B - \frac{1}{\sqrt{3}}}{1 + \left(\frac{1}{\sqrt{3}}\right)\tan B} = -\sqrt{3}$ So $\tan B - \frac{1}{\sqrt{3}} = -\sqrt{3} - \tan B$ So $2\tan B = -\frac{2}{\sqrt{3}}$, i.e. $\tan B = -\frac{1}{\sqrt{3}}$ Reference angle $= 30°$ In Quad's 2 or 4, so $B = 150°$ **or** $B = 330°$	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* • Reference angle $= 60°$ • Correct range for $B + 150°$ • Quadrants 2 and 4 identified • $\tan(A + B)$ formula with some substitution *Mid Partial Credit:* • $300°$ or $480°$ • Solution to equation outside of required range (for e.g., $B = -30°$ or $-210°$) • $\frac{\tan B + \tan 150}{1 - \tan B \tan 150} = -\sqrt{3}$ *High Partial Credit:* • $B = 150°$ or $B = 330°$ • $300°$ and $480°$

Q5	Model Solution – 30 Marks	Marking Notes
(a) (i)	$V_{sphere} = \frac{4}{3}\pi r^3$ $V_{cone} = \frac{1}{3}\pi r^2 h$ $V_{space} = \frac{4}{3}\pi r^3 - 2.\frac{1}{3}\pi r^2 h$ $= \frac{4}{3}\pi r^3 - \frac{2}{3}\pi r^2 r$ $= \frac{2}{3}\pi r^3$ $= \frac{1}{2} \times V_{sphere}$	**Scale 15D (0, 4, 8, 12, 15)** *Low Partial Credit:* • 2 volume formulas given (sphere and cone) • Volume formula given with some relevant manipulation / substitution *Mid Partial Credit:* • Volume of space in terms of r and h *High Partial Credit:* • Volume of space in terms of 1 variable *Full Credit -1:* • Incorrect or no conclusion, otherwise correct
(a) (ii)	$V_c = \frac{2}{3}\pi r^3 = \frac{686}{3}\pi$ $2r^3 = 686$ $r^3 = 343$ $r = 7$ cm	**Scale 5B (0, 2, 5)** *Partial Credit:* • $2.\frac{1}{3}\pi r^2 h = \frac{686}{3}\pi$ or equivalent, for example, $\frac{1}{3}\pi r^3 = \frac{686}{6}\pi$ *Full Credit –1* • Answer correct but incorrect or no units

<table>
<tr><th>Q5</th><th>Model Solution – 30 Marks</th><th>Marking Notes</th></tr>
<tr><td>(b)</td><td>

$60 \times 1{\cdot}75 = 105$ km travelled by van until $10{:}45$.

$105 + 60t = 95t$

So $t = 3$ hours

$10{:}45 + 3$ hr $= 13{:}45$

OR

$95 \times (t - 1{\cdot}75) = 60t$
$\Rightarrow\ t = 4\frac{3}{4}$ hours

$9{:}00 + 4\frac{3}{4}$ hr $= 13{:}45$

OR

Let distance travelled = x km

$$\frac{x}{60} - \frac{x}{95} = \frac{7}{4}$$

$x = 285$ km

$\Rightarrow\ t = \frac{285}{95} = 3$ hours

$10{:}45 + 3$ hr $= 13{:}45$

OR

$\frac{1{\cdot}75\times 60}{95-60} = 3$ hours.
$10{:}45 + 3 = 13{:}45$

OR

<table>
<tr><th>Time</th><th>Dist A (km)</th><th>Dist B (km)</th></tr>
<tr><td>$10{:}45$</td><td>$1{\cdot}75 \times 60$
$= 105$</td><td>0</td></tr>
<tr><td>$11{:}45$</td><td>$105 + 60$
$= 165$</td><td>95</td></tr>
<tr><td>$12{:}45$</td><td>$165 + 60$
$= 225$</td><td>$95 + 95$
$= 190$</td></tr>
<tr><td>$13{:}45$</td><td>$225 + 60$
$= 285$</td><td>$190 + 95$
$= 285$</td></tr>
</table>

Ans: $13{:}45$

</td><td>

Scale 10C (0, 3, 7, 10)

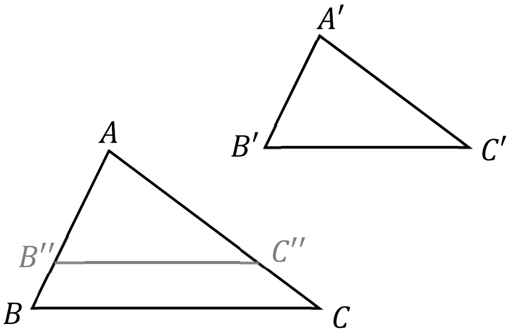
Low Partial Credit:

- Work of merit, for example:
  $1{\cdot}75$ or $\frac{7}{4}$ or 105 km;
  or $(t - 1{\cdot}75)$ or $(t + 1{\cdot}75)$;
  or $\frac{x}{60}$ or $\frac{x}{95}$

High Partial Credit:

- equation in t or in x, from which correct time / distance can be directly found
- correct table / diagram showing destination with all required distances / times indicated, but no conclusion
- $\frac{1{\cdot}75\times 60}{95-60}$

</td></tr>
</table>

Q6	Model Solution – 30 Marks	Marking Notes
(a)	**Step A (Given / Diagram)**  *Given*: ABC and $A'B'C'$ [similar triangles] [*To Prove*: $\frac{\|AB\|}{\|A'B'\|} = \frac{\|BC\|}{\|B'C'\|} = \frac{\|CA\|}{\|C'A'\|}$.] **Step B (Construction / Diagram):** *Construction*: Mark B'' on $[AB]$ such that $\|AB''\| = \|A'B'\|$. Mark C'' on $[AC]$ such that $\|AC''\| = \|A'C'\|$. [Join B'' to C''.] **Step C:** *Proof*: $\Delta\ AB''C''$ is congruent to $\Delta\ A'B'C'$ **Reason:** SAS **Step D:** $\therefore$ $B''C'' \|\| BC$ **Reason:** corresponding angles, $\|\angle\ AB''C''\| = \|\angle ABC\|$ **Step E:** $\therefore$ $\frac{\|AB\|}{\|AB''\|} = \frac{\|AC\|}{\|AC''\|}$ $\therefore$ $\frac{\|AB\|}{\|A'B'\|} = \frac{\|AC\|}{\|A'C'\|}$. Similarly, $\frac{\|AB\|}{\|A'B'\|} = \frac{\|BC\|}{\|B'C'\|}$. Hence, $\frac{\|AB\|}{\|A'B'\|} = \frac{\|BC\|}{\|B'C'\|} = \frac{\|CA\|}{\|C'A'\|}$.	**Scale 20D (0, 5, 10, 15, 20)** Consider the proof as requiring five steps, equivalent to those outlined in the model solution. Accept steps without reasons for up to *High Partial Credit*, but not for *Full Credit* Accept without last line if *To Prove* is filled in correctly. *Low Partial Credit:* • Work of merit, for example, relevant diagram(s) drawn, or effort at 'Given' *Mid Partial Credit:* • Any 2 steps *High Partial Credit:* • 4 steps presented

Q6	Model Solution – 30 Marks	Marking Notes
(b)	**S1.** $\lvert\angle HBQ\rvert = \lvert\angle HAP\rvert$ .... alternate **S2.** $\lvert\angle QHB\rvert = \lvert\angle PHA\rvert$ .... vertically opposite **S3.** So triangles are similar **S4.** So $\frac{\lvert AH\rvert}{\lvert HB\rvert} = \frac{\lvert AP\rvert}{\lvert QB\rvert}$ **S5.** So $\lvert AH\rvert \times \lvert QB\rvert = \lvert AP\rvert \times \lvert HB\rvert$	**Scale 10D (0, 3, 5, 8, 10)** *Note:* Step 5 is not considered done unless steps 1 to 4 are all present *Low Partial Credit:* • 1 relevant step listed or shown on diagram (no justification) • Mentions the relevant justifications *Mid Partial Credit:* • 3 relevant steps listed or shown on diagram (no justification) *High Partial Credit:* • All valid steps included but with no justification • 4 steps correct with at least one justification

Q7	Model Solution – 50 Marks	Marking Notes
(a)	$25 \times 1{\cdot}2 = 30$ $30 + 28 + 4 = 62$ [km]	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • $1{\cdot}2$ or $\frac{72}{60}$ *High Partial Credit:* • 30 km coming from $25 \times 1{\cdot}2$
(b)	$4{\cdot}8 - 1{\cdot}2 = 3{\cdot}6$ hours Let x be the speed at swimming, so: $T_{run} + T_{swim} = \frac{28}{5{\cdot}6x} + \frac{4}{x} = 3{\cdot}6$ $\frac{28+4(5{\cdot}6)}{5{\cdot}6x} = 3{\cdot}6$ $50{\cdot}4 = 20{\cdot}16x$ $x = 2{\cdot}5$ [km/h] **OR** $4{\cdot}8 - 1{\cdot}2 = 3{\cdot}6$ hours $T_{run} = \frac{28}{4} \times \frac{T_{swim}}{5{\cdot}6}$, so $T_{run} + T_{swim} = \left(\frac{28}{4} \times \frac{T_{swim}}{5{\cdot}6}\right) + T_{swim}$ $= \frac{9}{4} \times T_{swim} = 3{\cdot}6$ So $T_{swim} = \frac{8}{5}$ i.e. $\text{Speed}_{swim} = 4 \div \frac{8}{5} = 2{\cdot}5$ [km/h]	**Scale 15D (0, 4, 8, 12, 15)** *Low Partial Credit:* • Work of merit, for example: $4{\cdot}8 - 1{\cdot}2$ or $5{\cdot}6x$ or $\frac{28}{4}$ or $\frac{T_{swim}}{5{\cdot}6}$ *Mid Partial Credit:* • $\frac{4}{x}$ or $\frac{28}{5{\cdot}6x}$ or $\frac{5}{x}$ • $\frac{28}{4} \times \frac{T_{swim}}{5{\cdot}6}$ *High Partial Credit:* • An equation in one variable that can be solved to give a relevant time or speed
(c)	$30^2 = 28^2 + 4^2 - 2(28)(4)\cos C$ $\cos C = \frac{28^2+4^2-30^2}{2(28)(4)}$ $\cos C = -\frac{100}{224}$ $C = 116{\cdot}51 \ldots = 116{\cdot}5$ [1 D.P.]	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* • cosine rule formulated with some substitution *Mid Partial Credit:* • cosine rule formulated with full substitution *High Partial Credit:* • $\cos C = \frac{28^2+4^2-30^2}{2(28)(4)}$ or equivalent • $28^2 + 4^2 - 2(28)(4)\cos 116{\cdot}5°$ fully evaluated

Q7	Model Solution – 50 Marks	Marking Notes
(d)	$\text{Area} = \frac{1}{2} A\ B \sin C$ $\text{Area} = \frac{1}{2}(28)(4)\sin 116{\cdot}5°$ $= 50{\cdot}11 \ldots = 50{\cdot}1$ [km²] [1 D.P.] **Or** $\dfrac{30}{\sin 116{\cdot}5} = \dfrac{28}{\sin B}$ $\sin B = 0 \cdot 83527 \ldots$ $B = 56 \cdot 33^0$ $\text{Area} = \frac{1}{2}(30)(4)\sin 56{\cdot}33$ $= 50{\cdot}11 \ldots = 50{\cdot}1$ [km²] [1 D.P.]	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Area formula with some substitution *High Partial Credit:* • Area formula with full substitution
(e)	$\text{Area} = \frac{1}{2}$ base $\times\, d$ $50{\cdot}1 = \frac{1}{2}(30)d$ $d = \frac{50{\cdot}1}{15} = 3{\cdot}34 = 3{\cdot}3$ [km] [1 D.P.]	**Scale 10C (0, 3, 7, 10)** *Note:* divides $116{\cdot}5°$ by 2 and continues: award *Low Partial Credit* at most *Low Partial Credit:* • Indicates shortest distance with right angle (no credit awarded for this in (d)) • Equation with some substitution *High Partial Credit:* • Equation with full substitution
(f)	$\tan(0{\cdot}05) = \dfrac{x}{30}$ $30 \times \tan(0{\cdot}05) = x$ $x = 0{\cdot}02617 \ldots$ km $= 26$ [m] [$\in \mathbb{N}$]	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Relevant work on the diagram, for example, joins T to B and indicates angle of $0 \cdot 05°$ • Tan formula with some substitution *High Partial Credit:* • Tan formula with full substitution

<table>
<tr><th>Q8</th><th>Model Solution – 50 Marks</th><th>Marking Notes</th></tr>
<tr><td>(a)
(i)</td><td>Top 10% means 90% below.
$P(z < 1 \cdot 28) = 0{\cdot}8997.$
$\frac{x-176}{36} = 1{\cdot}28$
$\Rightarrow x = 222{\cdot}08$
Minimum mark of 223</td><td>Scale 10D (0, 3, 5, 8, 10)
Note: Accept use of $P(z < 1{\cdot}29)$, to give $x = 222{\cdot}44$
Note: Accept answer rounded to 222 instead of 223

Low Partial Credit:
• Mean or standard deviation indicated
• z-formula with some substitution

Mid Partial Credit:
• z-score found ($1{\cdot}28$ or $1{\cdot}29$)
• z-formula fully substituted $\left(\frac{x-176}{36}\right)$

High Partial Credit:
• $\frac{x-176}{36} = 1{\cdot}28$</td></tr>
<tr><td>(a)
(ii)</td><td>$P(165 < x < 210)$
$P\left(\frac{165-176}{36} < z < \frac{210-176}{36}\right)$
$= P(-0{\cdot}31 < z < 0{\cdot}94)$ [2 D.P.]
$= P(z < 0{\cdot}94) - P(z > -0{\cdot}31)$
$P(z < 0{\cdot}94) = 0{\cdot}8264$
$P(z < -0{\cdot}31) = 1 - P(z < 0{\cdot}31)$
$= 1 - 0{\cdot}6217 = 0{\cdot}3783$
So answer $= 0{\cdot}8264 - 0{\cdot}3783 = 0{\cdot}4481$
$= 44{\cdot}81\%$ of 1st years got the Distinction.</td><td>Scale 10D (0, 3, 5, 8, 10)
Note: Also accept use of $-0{\cdot}30$ instead of $-0{\cdot}31$, and/or use of $0{\cdot}95$ instead of $0{\cdot}94$.

Low Partial Credit:
• Mean or standard deviation indicated
• z formula with some substitution
• $-0{\cdot}305$ … or $0{\cdot}94$ …

Mid Partial Credit:
• One relevant probability found directly from tables ($0{\cdot}6217$ or $0{\cdot}8264$)

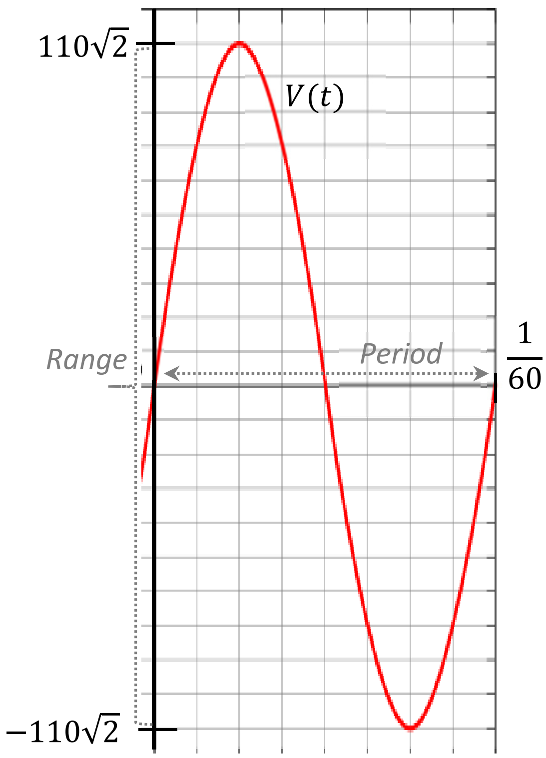
High Partial Credit:
• $0{\cdot}3783$ found
• $0{\cdot}8264$ and $0{\cdot}6217$ found

Full Credit –1:
• Uses $P(z > -0{\cdot}35)$ and finishes correctly</td></tr>
</table>

Q8	Model Solution – 50 Marks	Marking Notes
(b) (i)	$T = \frac{19{\cdot}8-21}{\left(\frac{5{\cdot}2}{\sqrt{60}}\right)} = -1{\cdot}787 \ldots$	**Scale 5C (0, 2, 3, 5)** *Note*: Accept $1{\cdot}787 \ldots$ *Note*: $\frac{s}{\sqrt{n}}$ must be used in order to be awarded above *Low Partial Credit* *Low Partial Credit:* • Mean or standard deviation indicated • Relevant formula with some substitution *High Partial Credit:* • Formula fully substituted
(b) (ii)	p-value: $p = 2[1 - P(z < 1{\cdot}79)]$ $= 2(1 - 0{\cdot}9633)$ $= 0{\cdot}0734$ Conclusion: There is **not** enough evidence to say that the claim in the news report is incorrect [as $0{\cdot}0734 > 0{\cdot}05$]	**Scale 10D (0, 3, 5, 8, 10)** *Note:* Accept $P(z < 1{\cdot}78)$, so p-value of $2(1 - 0{\cdot}9625) = 0{\cdot}075$. *Note:* Accept conclusion based on z-score rather than p-value. *Low Partial Credit:* • $P(z < 1{\cdot}79)$ • $0{\cdot}9633$ *Mid Partial Credit:* • $2[1 - P(z < 1{\cdot}79)]$ • Work of merit in finding p-value and correct conclusion based on this *High Partial Credit:* • p-value found but no or incorrect conclusion

Q8	Model Solution – 50 Marks	Marking Notes
(c) (i)	**Assuming no replacement:** $\frac{18}{23}\times\frac{17}{22}\times\frac{16}{21}\times\frac{5}{20}$ $= 0{\cdot}11518\ldots = 0{\cdot}1152$ [4 D.P.] **OR** **Assuming replacement:** $\left(\frac{18}{23}\right)^3\times\frac{5}{23}$ $= 0{\cdot}10420\ldots = 0{\cdot}1042$ [4 D.P.]	**Scale 10D (0, 3, 5, 8, 10)** *Note:* multiplication between relevant terms is necessary to be awarded above *Low Partial Credit* *Low Partial Credit:* • One relevant fraction *Mid Partial Credit:* • Product of four fractions, two of them correct *High Partial Credit:* • $\frac{18}{23}\times\frac{17}{22}\times\frac{16}{21}\times\frac{5}{20}$ • $\left(\frac{18}{23}\right)^3\times\frac{5}{23}$
(c) (ii)	$\left(\frac{12}{23}\times\frac{6}{22}\times\frac{5}{21}\right)3! = \frac{360}{1771}$ $= 0\cdot 20327\ldots = 0{\cdot}2033$ [4 D.P.] **OR** $\left(\frac{12}{23}\times\frac{6}{22}\times\frac{5}{21}\right)+\left(\frac{12}{23}\times\frac{5}{22}\times\frac{6}{21}\right)+$ $\left(\frac{6}{23}\times\frac{12}{22}\times\frac{5}{21}\right)+\left(\frac{6}{23}\times\frac{5}{22}\times\frac{12}{21}\right)+$ $\left(\frac{5}{23}\times\frac{6}{22}\times\frac{12}{21}\right)+\left(\frac{5}{23}\times\frac{12}{22}\times\frac{6}{21}\right)$ $= 0\cdot 20327\ldots = 0{\cdot}2033$ [4 D.P.] **OR** $\frac{\binom{12}{1}\times\binom{6}{1}\times\binom{5}{1}}{\binom{23}{3}} = 0{\cdot}2033$ [4 D.P.]	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • One relevant fraction, for example, $\frac{12}{23}$ or $\frac{6}{22}$ or $\frac{5}{23}$ • Counts / lists different possible arrangements, for example, 3! or 6 *High Partial Credit:* • $\frac{12}{23}\times\frac{6}{22}\times\frac{5}{21}$ or any other relevant triple • Assumes keys are replaced and finishes

Q9	Model Solution – 50 Marks	Marking Notes
(a) (i)	$\|\angle ABC\| = 180 - 20 = 160°$ $\frac{1450}{\sin 160} = \frac{x}{\sin 8{\cdot}57}$ $x = \frac{1450 \times \sin 8{\cdot}57}{\sin 160}$ $x = 631{\cdot}7626 \ldots$ Time $= \frac{631{\cdot}7626\ldots}{420} = 1{\cdot}504 \ldots$ hours $= 90$ mins **or** $1{\cdot}5$ hours **or** 1 hour 30 mins **OR** $\|AB\| = 2 \times 420 = 840$ km $\|BC\|^2 = 1450^2 + 840^2 - 2(1450)(840)\cos 8 \cdot 57$ $\|BC\|^2 = 399299{\cdot}05 \ldots$ $\|BC\| = 631{\cdot}90 \ldots$ $\frac{631{\cdot}90\ldots}{420} = 1{\cdot}50 \ldots$ hours etc	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* • sine rule or cosine rule stated with some substitution • finds $\|AB\|$ or $\|\angle ABC\|$ *Mid Partial Credit:* • sine rule or cosine rule with full substitution *High Partial Credit:* • $\|BC\|$ (that is, x) found
(a) (ii)	$\text{Time}_{[AC]} = \frac{1450}{420} = 3{\cdot}4523 \ldots$ hours Total time $= 2 + 1{\cdot}5 + 3{\cdot}4523 \ldots$ $= 6{\cdot}9523 \ldots$ hours $= 25\ 028{\cdot}57 \ldots$ seconds Max possible flight time $= \frac{100\ 000}{3{\cdot}8}$ $= 26\ 315{\cdot}7 \ldots$ seconds, which is greater than $25\ 028{\cdot}57 \ldots$ sec **OR** *Finds* $25\ 028{\cdot}57 \ldots$ seconds Litres required $= 3{\cdot}8 \times 25\ 028{\cdot}57 \ldots$ $= 95\ 108 \cdot 57 \ldots$ litres, which is less than $100\ 000$ litres.	**Scale 10D (0, 3, 5, 8, 10)** Consider solution as requiring four steps, equivalent to: **1.** Finds time in hours to travel $[AC]$ **2.** Finds total time in hours (3 sides) **3.** Converts total time to seconds **4.** Converts time (sec) to litres required *Low Partial Credit:* • One relevant calculation, for example, $\frac{1450}{420}$ or $2 + 1{\cdot}5$ or $\frac{100\ 000}{3{\cdot}8}$ *Mid Partial Credit:* • Two steps *High Partial Credit:* • Three steps

Q9	Model Solution – 50 Marks	Marking Notes
(b) (i)	Range: $\left[-110\sqrt{2},\ 110\sqrt{2}\right]$ Period: $\frac{2\pi}{120\pi}$ or $\frac{1}{60}$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Work of merit, for example, some indication of the period of a sine function; some mention of $110\sqrt{2}$ *High Partial Credit* • Period or range correct *Full Credit –1:* • Apply a * for period and range swapped
(b) (ii)		**Scale 10C (0, 3, 7, 10)** *Note:* Consider solution as requiring 3 aspects: **1.** sine curve of at least one period, including $(0, 0)$ **2.** range indicated **3.** period indicated *Low Partial Credit:* • Period or range from **(b)(i)** indicated on axes, but no or incorrect graph • Graph a recognisable portion of a sine curve, or similar *High Partial Credit:* • Two of the aspects above present on the graph
(b) (iii)	$V(6{\cdot}67) = 110\sqrt{2}\,\sin 120\pi 6{\cdot}67$ $= 147{\cdot}949\ldots = 147{\cdot}95$ [Volts] [2 D.P.]	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Formula with some substitution *High Partial Credit:* • Formula with full substitution *Full Credit –1:* • Calculator in incorrect mode

Q9	Model Solution – 50 Marks	Marking Notes
(b) (iv)	*Accept any value of t satisfying* $t = \frac{1+8n}{480}$ *or* $t = \frac{3+8n}{480}$*, as long as in the correct form.* $V(t) = 110\sqrt{2}\,\sin 120\pi t = 110$ So $\sin 120\pi t = \frac{1}{\sqrt{2}}$ So $120\pi t = \frac{\pi}{4}$, i.e. $t = \frac{1}{480}$ [seconds] **or** $120\pi t = \frac{3\pi}{4}$ => $t = \frac{3}{480}$ [seconds]	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit:* • Equation with some substitution *High Partial Credit:* • Equation with full substitution *Full Credit –1:* • Calculator in incorrect mode
(b) (v)	$V(t) = 110\sqrt{2}\,\sin(120\pi t)$ $V'(t) = 120\pi \times 110\sqrt{2}\,\cos(120\pi t)$ $V'(2) = 120\pi \times 110\sqrt{2}\,\cos(120\pi \times 2)$ $= 58646{\cdot}0 = 58646$ Volts /sec [$\in \mathbb{N}$]	**Scale 5C (0, 2, 3, 5)** *Note:* $V'(t)$ must be correct in order to be awarded more than *Low Partial Credit*. *Low Partial Credit:* • any correct differentiation *High Partial Credit:* • Expression for $V'(t)$ *Full Credit –1* • Calculator in incorrect mode • Correct answer with no or incorrect units

Q10	Model Solution – 50 Marks	Marking Notes
(a) (i)	$P(2 \text{ from the 1st 9 are O}^-) \times P(10\text{th is O}^-)$ $= \binom{9}{2}\left(\frac{8}{100}\right)^2\left(\frac{92}{100}\right)^7 \times \frac{8}{100}$ $= 0{\cdot}01028 \ldots = 0{\cdot}0103$ [4 D.P.]	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* • $\frac{8}{100}$ or $\frac{92}{100}$ or $\binom{9}{2}$ • First line of solution indicated (accept with "and" instead of $\times$) *Mid Partial Credit:* • $\binom{9}{2}\left(\frac{8}{100}\right)^2\left(\frac{92}{100}\right)^7$ • $\binom{10}{3}\left(\frac{8}{100}\right)^3\left(\frac{92}{100}\right)^7$ evaluated *High Partial Credit:* • $\binom{9}{2}\left(\frac{8}{100}\right)^2\left(\frac{92}{100}\right)^7\frac{8}{100}$ • $\binom{10}{3}\left(\frac{8}{100}\right)^3\left(\frac{92}{100}\right)^7\frac{8}{100}$ evaluated
(a) (ii)	$1 - P(\text{none are O}^-)$ $= 1 - \left(\frac{92}{100}\right)^5$ $= 0{\cdot}34091 \ldots = 0{\cdot}3409$ [4 D.P.] **OR** $P(1 \text{ or } 2 \text{ or } 3 \text{ or } 4 \text{ or } 5 \text{ are O}^-)$ $= \binom{5}{1}\left(\frac{8}{100}\right)^1\left(\frac{92}{100}\right)^4 + \binom{5}{2}\left(\frac{8}{100}\right)^2\left(\frac{92}{100}\right)^3$ $+\binom{5}{3}\left(\frac{8}{100}\right)^3\left(\frac{92}{100}\right)^2 + \binom{5}{4}\left(\frac{8}{100}\right)^4\left(\frac{92}{100}\right)^1$ $+\left(\frac{8}{100}\right)^5$ $= 0{\cdot}34091 \ldots = 0{\cdot}3409$ [4 D.P.]	**Scale 10C (0, 3, 7, 10)** *Low Partial Credit:* • $\left(\frac{92}{100}\right)^a$ where $0 < a < 5$ • First line of either solution *High Partial Credit:* • $\left(\frac{92}{100}\right)^5$ • Three terms in second solution
(a) (iii)	$1 - 0{\cdot}92^k > 0{\cdot}97$ so $0{\cdot}92^k < 0{\cdot}03$ Find where $0{\cdot}92^k = 0{\cdot}03$ i.e. $k(\ln(0{\cdot}92)) = \ln(0{\cdot}03)$ so $k = \frac{\ln(0{\cdot}03)}{\ln(0{\cdot}92)} = 42{\cdot}05 \ldots$ so least $k = 43$	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* • $0{\cdot}92^k$ • $0{\cdot}03$ *Mid Partial Credit:* • $1 - 0{\cdot}92^k > 0{\cdot}97$ or $= 0{\cdot}97$ • $k(\ln(0{\cdot}92))$ *High Partial Credit:* • Equation in k without indices (logs handled correctly)

Q10	Model Solution – 50 Marks	Marking Notes
(b)	**Interpretation 1:** **initial €70 charged regardless** $0{\cdot}8(70) + 0{\cdot}2(70 + 150 + 80)$ $= 0{\cdot}8(70) + 0{\cdot}2(300) = €116$ **Interpretation 2:** **initial €70 not charged if not successful** $0{\cdot}8(70) + 0{\cdot}2(150 + 80)$ $= 0{\cdot}8(70) + 0{\cdot}2(230) = €102$	**Scale 10C (0, 3, 7, 10)** *Low Partial Credit:* • A correct calculation, for example, $0{\cdot}8(70)$ or 300 or 230 *High Partial Credit:* • $0{\cdot}8(70)$ and $0{\cdot}2(300)$ **or** $0{\cdot}8(70)$ and $0{\cdot}2(230)$
(c)	Average pay-out per customer: $120\,000(0{\cdot}0001) + 40\,000(0{\cdot}002)$ $= €92$ Target profit per customer: $\frac{900\,000}{18\,000} = €50$ Required premium: $50 + 92 = €142$ **OR** Average pay-out per customer: $120\,000(0{\cdot}0001) + 40\,000(0{\cdot}002)$ $= €92$ Total expected payout: $€92 \times 18\,000 = €1\,656\,000$ Total revenue required: $€1\,656\,000 + €900\,000 = €2\,556\,000$ Required premium: $€2\,556\,000 \div 18\,000 = €142$	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* • One relevant calculation, for example, $120\,000(0{\cdot}0001)$ or $\frac{900\,000}{18\,000}$ *Mid Partial Credit:* • Finds €92 (average payout per person) *High Partial Credit:* • Finds €2 556 000 (total revenue) • Finds €92 and €50

Marcanna Breise as ucht freagairt trí Ghaeilge

Léiríonn an tábla thíos an méid marcanna breise ba chóir a bhronnadh ar iarrthóirí a ghnóthaíonn níos mó ná 75% d'iomlán na marcanna.

N.B. Ba chóir marcanna de réir an ghnáthráta a bhronnadh ar iarrthóirí nach ngnóthaíonn níos mó ná 75% d'iomlán na marcanna don scrúdú. Ba chóir freisin an marc bónais sin **a shlánú síos**.

Tábla 220 @ 5%
Bain úsáid as an tábla seo i gcás na n-ábhar a bhfuil 220 marc san iomlán ag gabháil leo agus inarb é 5% gnáthráta an bhónais.

Bain úsáid as an ngnáthráta i gcás 165 marc agus faoina bhun sin. Os cionn an mharc sin, féach an tábla thíos.

Bunmharc	Marc Bónais
166	8
167 - 173	7
174 - 180	6
181 - 186	5
187 - 193	4

Bunmharc	Marc Bónais
194 - 200	3
201 - 206	2
207 - 213	1
214 - 220	0