chapter

10 Patterns and Sequences

Key words

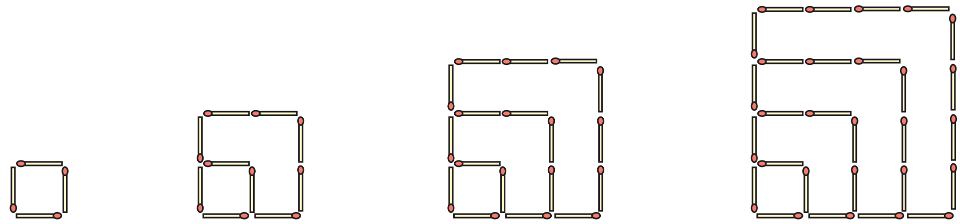
pattern **sequence** **term** **method of difference** ***n*th term**
coefficient **arithmetic sequence** **common difference** **arithmetic series**
S_n, the sum to *n* terms **quadratic sequence**

Section 10.1 Patterns in number

The ability to see patterns or sequences is very important in mathematics.
We meet number patterns such as 1, 3, 5, 7, … or 5, 10, 15, 20, … on a regular basis.

We see **patterns** in designs such as tiling and mosaics.

Here is a growth pattern of squares made from matchsticks.



pattern 1 4 | pattern 2 10 | pattern 3 18 | pattern 4 28

The numbers in red below each pattern represent the numbers of matches used in each shape.
The numbers 4, 10, 18, 28, … generated by these patterns form a sequence that is a little more complex than the sequences 1, 3, 5, 7, … or 5, 10, 15, 20, … .

Number sequences

A number sequence is an ordered set of numbers with a rule to find every number in the sequence. The rule which takes you from one number to the next could be a simple addition or multiplication, but generally it is more tricky than that. In more difficult sequences you need to examine them carefully to identify the pattern.

Each number in a sequence is called a **term**.
The first term is written as T_1 ; the 4th term is T_4.

Look at these sequences and their rules.

4, 8, 16, 32, … doubling the preceding term each time … 64, 128, …
4, 7, 10, 13, … adding 3 to the preceding term each time … 16, 19, …
36, 32, 28, 24, … subtracting 4 from the preceding term each time … 20, 16, …

These sequences are all quite straightforward once you have found the link from one term to the next.

Method of differences

For some sequences that are not immediately obvious, we need to look at the difference between consecutive terms to determine the pattern.

Consider the sequence:	3		6		11		18		27
Difference between terms:		3		5		7		9	

Here the differences form a sequence of their own.
The pattern is much easier to detect in this sequence.
The next difference is 11.
We now find the next term of the first sequence by adding 11 to 27.
The next term is $27 + 11$, i.e., 38.

Exercise 10.1

1. Look at the following number sequences.
Write down the next three terms in each and explain how the sequence is found.

(i) 2, 4, 6, 8, … (ii) 1, 3, 5, 7, …
(iii) 1, 4, 7, 10, … (iv) 1, 2, 4, 8, …
(v) 3, 9, 27, … (vi) 16, 8, 4, …
(vii) 20, 18, 16, … (viii) 2, 6, 18, …

2. By considering the differences in the following sequences, write down the next two terms in each case:

(i) 2, 4, 7, 11, … (ii) 1, 2, 5, 10, …
(iii) 2, 6, 12, 20, … (iv) 2, 3, 6, 11, 18, …
(v) 1, 4, 10, 19, … (vi) 2, 7, 14, 23, …

3. Look carefully at each number sequence below.
Find the next two terms in the sequence and try to explain the pattern.

(i) 1, 1, 2, 3, 5, 8, 13, … (ii) 3, 4, 7, 11, 18, 29, …
(iii) 1, 8, 27, 64, … (iv) $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots$

4. Examine these number patterns:
Write down the next line in each pattern without using a calculator.
Now use a calculator to check that you are correct.

(i)	(ii)	(iii)
$6 \times 9 = 54$	$9 \times 1 = 9$	$7 \times 7 = 49$
$66 \times 9 = 594$	$9 \times 12 = 108$	$67 \times 67 = 4489$
$666 \times 9 = 5994$	$9 \times 123 = 1107$	$667 \times 667 = 444889$
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Section 10.2 The *n*th term of a sequence

When using a number sequence, we sometimes need to know, for example, the 50th or 100th term without having to write out all 50 or 100 terms. To do this we need to find the rule which generates the sequence.

This rule is generally called the ***n*th** term or T_n.

If $T_n = 2n + 3$, then $T_1 = 2(1) + 3 = 5$

$T_2 = 2(2) + 3 = 7$

$T_3 = 2(3) + 3 = 9$

....

From this we can see that $T_n = 2n + 3$ generates the sequence 5, 7, 9, …

The rule, $T_n = 2n + 3$, allows us to find any term of the sequence.

Example 1

The *n*th term of a sequence is $4n - 3$.
Write down the first five terms of the sequence.

$T_n = 4n - 3$

$T_1 = 4(1) - 3 = 1$

$T_2 = 4(2) - 3 = 5$

$T_3 = 4(3) - 3 = 9$

$T_4 = 4(4) - 3 = 13$

$T_5 = 4(5) - 3 = 17$

The first 5 terms are: 1, 5, 9, 13, 17.

Finding the *n*th term of a sequence

Consider the sequence 2, 5, 8, 11, …
Here we have the **same difference** between any term and the next.
This difference is **3**.
The nth term of the sequence will be $\mathbf{3}n \pm$ a number.
$3n$ will generate the sequence 3, 6, 9, 12.