

2. Given $z = 2\left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right)$, find z^2 in the form $r(\cos \theta + i \sin \theta)$. ✓

(3.8)

If $z_1 = r_1(\cos \theta_1 + i \sin \theta_1)$
 and $z_2 = r_2(\cos \theta_2 + i \sin \theta_2)$,
 then $* z_1 z_2 = r_1 r_2 [\cos(\theta_1 + \theta_2) + i \sin(\theta_1 + \theta_2)]$
 and $\frac{z_1}{z_2} = \frac{r_1}{r_2} [\cos(\theta_1 - \theta_2) + i \sin(\theta_1 - \theta_2)]$.

$$r = 2$$

$$\theta = \frac{\pi}{3}$$

$$z^2 = (2)(2) \left[\cos\left(\frac{\pi}{3} + \frac{\pi}{3}\right) + i \sin\left(\frac{\pi}{3} + \frac{\pi}{3}\right) \right]$$

$$= 4 \left[\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} \right] \checkmark$$

De Moivre's theorem

$(\cos \theta + i \sin \theta)^n \equiv \cos n\theta + i \sin n\theta$,
 for all real values of n .

$$\text{eg... } \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right)^8$$

$$= \cos 8\left(\frac{\pi}{4}\right) + i \sin 8\left(\frac{\pi}{4}\right)$$

$$= \cos 2\pi + i \sin 2\pi$$

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If $z = r(\cos \theta + i \sin \theta)$, then using de Moivre's Theorem:

$$\begin{aligned} z^n &= [r(\cos \theta + i \sin \theta)]^n \\ &= r^n(\cos n\theta + i \sin n\theta) \text{ for all } n \in \mathbb{Z}. \end{aligned}$$

Example 1

Find the value of $(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6})^3$.

de Moivre

$$= \cos \frac{3\pi}{6} + i \sin \frac{3\pi}{6}$$

$$= \cos \frac{\pi}{2} + i \sin \frac{\pi}{2}$$