9. On his way to work, Nick goes through a set of traffic lights and then passes over a level crossing.
Over a period of time, Nick has estimated the probability of stopping at each of these.

The probability that he has to stop at the traffic lights is $\frac{2}{3}$.
The probability that he has to stop at the level crossing is $\frac{1}{5}$.

These probabilities are independent.

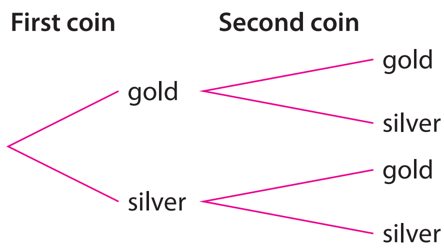
(i) Construct a tree diagram to show this information.
(ii) Calculate the probability that Nick will not have to stop at either the lights or the level crossing on his way to work.

10. An ordinary coin is tossed three times.
Draw a tree diagram to show all the possible outcomes.
Work out the probability of getting

(i) 3 heads
(ii) 2 heads and a tail in any order.

11. A bag contains 20 coins.
There are 6 gold coins and the rest are silver.
A coin is taken at random from the bag.
The type of coin is recorded and the coin is then returned to the bag.
A second coin is then taken at random from the bag.

(i) The tree diagram shows all the ways in which two coins can be taken from the bag.
Copy the diagram and write the probabilities on it.
(ii) Use your tree diagram to calculate the probability that one coin is gold and one coin is silver.



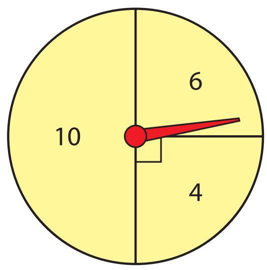
Section 6.9 Expected value

The circle shown is divided into 3 sectors.

When the spinner is spun it will land on 10, 6 or 4.

If the spinner is spun twice and we get 10 and 6, then the average of the two spins is $\frac{10 + 6}{2}$ i.e. 8.

If the spinner is spun 100 times, is there a quick way of finding the 'average' value of these spins?



This average or **expected value** is found by multiplying each number by its probability and adding the results.

This is set out in the table below.

Outcome (x)	Probability (P)	$x \times P$
10	$\frac{1}{2}$	5
6	$\frac{1}{4}$	$1\frac{1}{2}$
4	$\frac{1}{4}$	1

When each outcome is multiplied by the corresponding probability, we get 5, $1\frac{1}{2}$ and 1.

The sum of these results is $7\frac{1}{2}$.

The number $7\frac{1}{2}$ is the **expected value**.

If the spinner above is spun a large number of times, the mean value of the outcomes approaches the expected value $7\frac{1}{2}$. Statisticians call this fact the **law of large numbers**.

Notice that the expected value $7\frac{1}{2}$ is not one of the outcomes 4, 6 and 10.

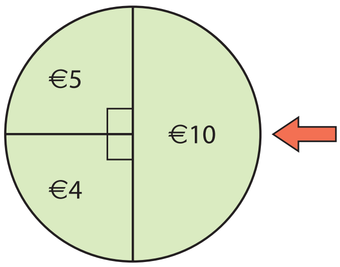
In general, the expected value need not be one of the given outcomes.

The expected value of the outcome of an experiment is denoted by **E(x)**.
When all the outcomes are multiplied by their corresponding probabilities and the results added, the operation can be expressed in a concise way as follows:

$E(x) = \Sigma x.P(x)$, where Σ represents 'the sum of'.

The expected value is widely used in the insurance industry and in the operation of casino games. If you would like to know whether or not a casino game is fair, you would need to know what the payout is and the probability of getting that payout. In simple terms, you need to know the expected value of the payouts.

Let us consider this fun-park spinning-wheel on the right.
It costs €8 to spin the wheel and you win the amount to which the arrow is pointing.
Is this a "fair game?"



First we calculate the expected value of the payout.

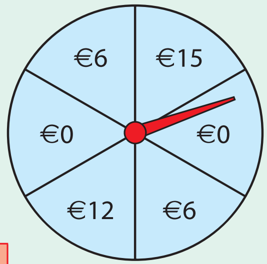
Outcome (x)	Probability (P)	$x \times P$
€10	$\frac{1}{2}$	€5
€5	$\frac{1}{4}$	€1.25
€4	$\frac{1}{4}$	€1

$\Sigma x.P(x) = €5 + €1.25 + €1 = €7.25$

The expected value of the payout is €7.25.
But it costs €8 to spin the wheel.
The expected payout now is €7.25 − €8, i.e., −€0.75
Thus if the wheel is spun a large number of times you could expect to lose €0.75 on average on each spin.
To determine whether or not a game is fair, we need to take into account
(i) the expected value of the payout
(ii) the cost of playing the game.
Then (a) if the expected payout is zero, the game is fair
(b) if the expected payout is greater than zero, you will win in the long-run
(c) if the expected payout is less than zero, you will lose in the long-run.

Example 1

This circle is divided into 6 equal sectors.
You pay €8 to spin the arrow and you win the amount in the sector in which the arrow stops.
What is the expected amount you win or lose in this game?



We find the expected value of the payout as follows:

Payout (x)	Probability (P)	Payout × Probability
0	$\frac{2}{6}$	0
6	$\frac{2}{6}$	€2
12	$\frac{1}{6}$	€2
15	$\frac{1}{6}$	€2.50

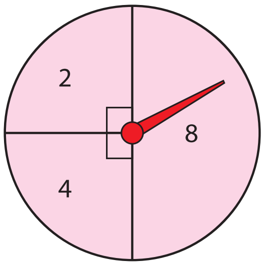
$\Sigma x.P(x)$ = €0 + €2 + €2 + €2.50 = €6.50
The expected payout is €6.50.
When you pay the €8 to play, the expected payout is
€6.50 − €8 = −€1.50
So you can expect to lose €1.50 if you play the game.

Exercise 6.9

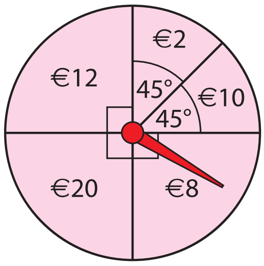
1. The table on the right shows the outcomes and their probabilities when a fair dice is thrown.
Copy and complete the table and show that the expected value is 3.5.

Outcome (x)	Probability (P)	$x \times P$
1	$\frac{1}{6}$	$\frac{1}{6}$
2	$\frac{1}{6}$	
3	$\frac{1}{6}$	
4	$\frac{1}{6}$	
5	$\frac{1}{6}$	$\frac{5}{6}$
6	$\frac{1}{6}$	

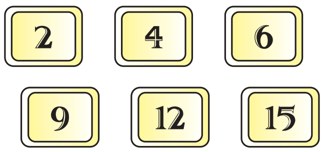
2. Find the expected value when this spinner is spun a large number of times.



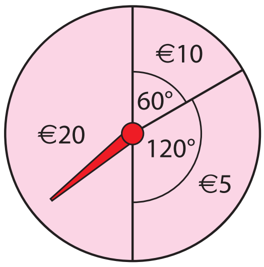
3. When this spinner is spun the amount in the sector in which the arrow stops is paid out.
What is the expected value of the payout.



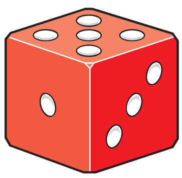
4. A card is selected at random from the cards shown and then replaced. The process is repeated several times.
Find the expected value of the number selected.



5. In the given wheel, you win the amount in the sector in which the arrow stops.
It costs €10 to play the game. How much could you expect to win or lose if you play this game?
Explain why the game is not fair.



6. In a casino, it costs €6 to throw a dice.
If you roll a 3, you win €12.
If you roll an even number, you win €6.
For the remaining numbers, you don't win anything.
How much can you expect to win or lose if you play this game?



7. A card is drawn from a normal pack of cards.
If a king is drawn you win €50.
If a diamond is drawn you win €8.
If a Jack is drawn you lose €5.
If you draw any other card, you neither win nor lose.
If it costs €10 to play this game, how much can you expect to win or lose?
Give your answer correct to the nearest 10c.

8. A sports club sells 1000 tickets for a confined draw.
There is one prize of €100, five prizes of €50 and ten prizes of €20.
Find the expected value of a prize.

9. Here is the sample space when two dice are thrown and the scores are added.

(i) What is the probability of getting a total of 9?

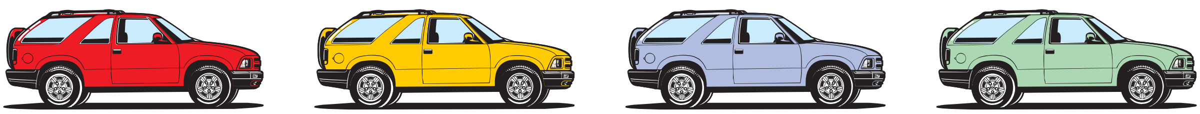
In a casino, a game consists of throwing two dice and adding the scores.
If you score a total of 7, you win €24.
If you score 9, you lose €27.
For all other scores you neither win nor lose.
If you play this game, what do you expect to win or lose?
If you pay €2 to play this game, could you say it was a fair game? Explain you answer.

	1	**2**	**3**	**4**	**5**	**6**
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

10. Make out a sample space for all the outcomes when three coins are tossed.
A game consists of tossing three coins and counting the number of heads obtained.
If you get exactly two heads you win €20.
For any other result you lose €5.
If you pay €2 to play this game, how much can you expect to win or lose?
Use your answer to state whether or not the game is fair.

Section 6.10 The fundamental principle of counting

A make of car comes in four different models as shown below:



Standard (S) Classic (C) Elegant (E) Diamond (D)

Each model comes in three different colours: silver(s), red(r), and black(b).
Here are the choices a customer has:

$(S, s), (S, r), (S, b), (C, s), (C, r), (C, b), (E, s), (E, r), (E, b), (D, s), (D, r), (D, b).$