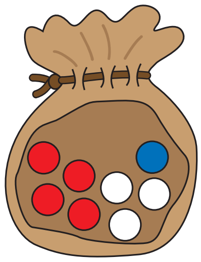
Exercise 6.4

1. A fair coin is tossed 100 times.
How many heads would you expect to get?

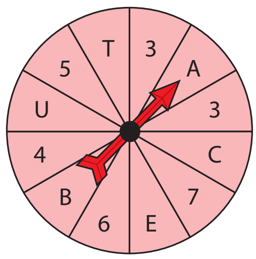
2. A fair six-sided dice is thrown 60 times.
(i) How many sixes would you expect to get?
(ii) How many twos would you expect to get?
(iii) How many twos or sixes would you expect to get?

3. One ball is selected at random from the bag shown and then replaced. This procedure is done 400 times.
How many times would you expect to select:
(i) a blue ball,
(ii) a white ball?



4. Joe thinks his coin is biased.
He tosses it 200 times and gets 130 heads and 70 tails.
(i) What is the experimental probability of getting a head with this coin?
(ii) In 200 tosses, how many heads would you expect to get if the coin was fair?
(iii) Do you think the coin is biased?
Explain your answer.

5. A spinner, with 12 equal sectors, is spun 420 times.
How often would you expect to spin:
(i) an E
(ii) an even number
(iii) a vowel?

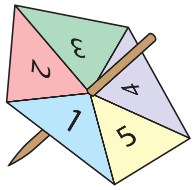


6. Helen wanted to find out if a dice was biased. She threw the dice 300 times.
Her results are given in the table below.

Number on dice	1	2	3	4	5	6
Frequency	30	40	55	65	50	60

(i) For this dice, calculate the experimental probability of obtaining
(a) a 6 (b) a 2.
(ii) For a fair dice, calculate the probability of scoring
(a) a 6 (b) a 2.
(iii) Do your answers suggest that the dice is fair?
Give your reasons.

7. A spinner is labelled as shown.
The results of the first 30 spins are given below.

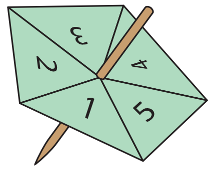


1	2	3	3	5	1	3	2	2	4	5	3	2	1	2
5	2	4	1	5	1	5	2	2	4	2	5	4	2	3

Construct a table showing the number of ones, twos, etc. scored.
If the spinner was fair, how many times would you expect each number to appear?
Do you think this spinner is fair? Give a reason for your answer.

8. Gemma keeps a record of her chess games with Helen.
Out of the first 10 games, Gemma wins 6.
Out of the first 30 games, Gemma wins 21.
Based on these results, estimate the probability that Gemma will win her next game of chess with Helen.

9. This spinner is biased.
The probability that the spinner will land on each of the numbers 1 to 4 is given in the table below.



Number	1	2	3	4	5
Probability	0.35	0.1	0.25	0.15	

The spinner is spun once.
(i) Work out the probability that the spinner will land on 5.
(ii) Write down the number on which the spinner is most likely to land.
(iii) If the spinner is spun 200 times, how many times would you expect it to land on 3?

10. Olivia, Ben and Joe each rolled a different dice 360 times.

Only one of the dice was fair.
Whose was it?
Explain your answer.
Whose dice is the most biased?
Explain your answer.

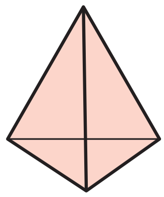
Number	Olivia	Ben	Joe
1	27	58	141
2	69	62	52
3	78	63	56
4	43	57	53
5	76	56	53
6	67	64	5

11. The probability that a biased dice will land on each of the numbers 1 to 6 is given in the table below:

Number	1	2	3	4	5	6
Probability	x	0.2	0.1	0.3	0.1	0.2

(i) Calculate the value of x.
(ii) If the dice is thrown once, find the probability that the dice will show a number higher than 3.
(iii) If the dice is thrown 1000 times, estimate the number of times it will show a 6.

12. A four-sided dice has faces numbered 1, 2, 3 and 4. The 'score' is the number on which it lands. Five pupils throw the dice to see if it is biased. They each throw it a different number of times. Their results are shown in the table.



Pupil	Total number of throws	Score			
		1	2	3	4
Andy	20	7	6	3	4
Brian	50	19	16	8	7
Ciara	250	102	76	42	30
Dara	80	25	25	12	18
Emma	150	61	46	26	17

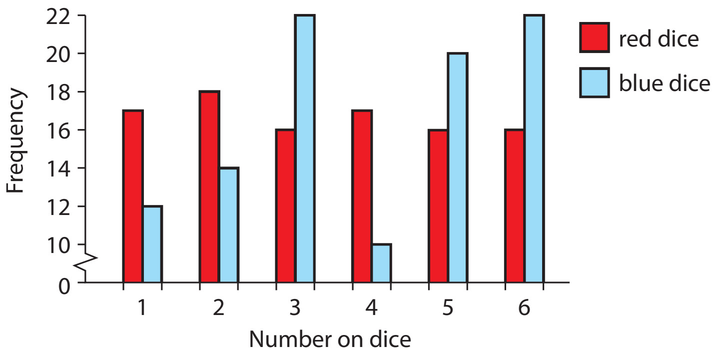
(i) Which pupil will have the most reliable set of results? Why?

(ii) Add up all the score columns and work out the relative frequency of each score. Give your answers to one decimal place.

(iii) Is the dice biased? Explain your answer.

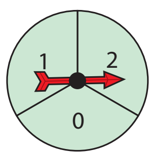
13. A red and blue dice were each tossed 100 times.

This bar chart shows the results.
One of the dice is fair and the other unfair. Which do you think is the fair dice? Why?



14. Four friends are using a spinner for a game and they wonder if it is perfectly fair. They each spin the spinner several times and record the results.

Name	Number of spins	Results		
		0	1	2
Alan	30	12	12	6
Keith	100	31	49	20
Bill	300	99	133	68
Ann	150	45	73	32



(i) Whose results are most likely to give the best estimate of the probability of getting each number?

(ii) Make a table by adding together all the results.
Use the table to decide whether you think the spinner is biased or unbiased.

(iii) Use the results to work out the probability of the spinner getting a '2'.

15. There are 100 sweets in a box.
Eric takes a sweet without looking.
He writes down what sort it is and then **puts it back**.
He does this 100 times. This chart shows Eric's results.

toffee	20
mint	38
jelly	14
choco	25
caramel	3

(i) Eric thought there must be exactly 20 toffees in the box. Explain why he is wrong.
(ii) What is the smallest number of caramels that could be in the box?
(iii) Is it possible there is any other sort of sweet in the box? Explain.
(iv) Eric starts again and does the same thing another 100 times. Will his chart have exactly the same numbers on it?
(v) Eric's friend takes a sweet. What sort is he most likely to get?

Section 6.5 Mutually exclusive events – the addition rule

Seven cards with different numbers and colours are shown:

Consider these two events:
(i) drawing a red card (ii) drawing an even number.

These two outcomes cannot happen together as there is no red card with an even number on it.

These outcomes are said to be **mutually exclusive**.

If the events A and B cannot happen together, then

$$P(\text{A or B}) = P(\text{A}) + P(\text{B})$$

This is called the **addition law** for mutually exclusive events.

> Outcomes are mutually exclusive if they cannot happen at the same time.

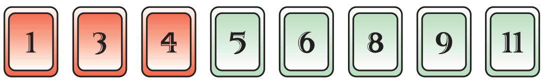
Using the cards above,

$$P(\text{red card or even number}) = P(\text{red card}) + P(\text{even number})$$
$$= \tfrac{3}{7} + \tfrac{2}{7} = \tfrac{5}{7}$$

> The addition law is sometimes called the **OR** Rule

When events are not mutually exclusive

Now consider these cards:



What is the probability of a red card or an even number?
There are 3 red cards and 3 even numbers.

The probability is not $P(\text{red card}) + P(\text{even number})$
i.e., not $\frac{3}{8} + \frac{3}{8}$ as the number 4 is counted twice.

There are only 5 red cards or even numbers.

$\therefore \quad P(\text{red card or even number}) = \frac{5}{8}$

For the cards on the previous page,

P(red or even number) $= P$(red) $+ P$(even number) $- P$(red and even number)

$= \frac{3}{8} + \frac{3}{8} - \frac{1}{8}$

$= \frac{6}{8} - \frac{1}{8} = \frac{5}{8}$

In general when two events A and B can occur at the same time,

***P*(A or B) = *P*(A) + *P*(B) − *P*(A and B)**

Example 1

A number is to be selected at random from the integers 1 to 30 inclusive. Find the probability that the number is

(i) a multiple of 3
(ii) a multiple of 5
(iii) a multiple of 3 or a multiple of 5.

(i) The multiples of 3 are: 3, 6, 9, 12, 15, 18, 21, 24, 27, 30.

$\Rightarrow \quad P(\text{multiple of } 3) = \frac{10}{30} = \frac{1}{3}$

(ii) The multiples of 5 are: 5, 10, 15, 20, 25, 30.

$\Rightarrow \quad P(\text{multiple of } 5) = \frac{6}{30} = \frac{1}{5}$

(iii) The numbers 15 and 30 are each multiples of both 3 and 5.

$\Rightarrow \quad P(\text{multiple of 3 or 5}) = P(\text{multiple of } 3) + P(\text{multiple of } 5)$

$- P(\text{multiple of 3 and 5})$

$= \frac{10}{30} + \frac{6}{30} - \frac{2}{30} = \frac{14}{30} = \frac{7}{15}$

Exercise 6.5

1. An unbiased dice is thrown.
Find the probability that the number showing is
(i) 3 (ii) an even number (iii) a 3 or an even number.

2. A box contains discs numbered 1 to 16.
If a disc is selected at random, what is the probability that it is
(i) an odd number (ii) a multiple of 4
(iii) an odd number or a multiple of 4?

3. A bag contains 4 red, 3 blue and 2 green marbles.
If a marble is selected at random, what is the probability that it is
(i) a red marble (ii) a green marble (iii) a red or a green marble?

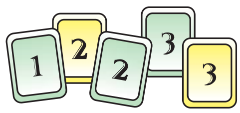
4. A card is selected at random from a pack of 52 playing cards.
What is the probability that it is
(i) a spade (ii) a red picture card
(iii) a spade or a red picture card?

5. A number is selected at random from the integers 1 to 12 inclusive.
Find the probability that the number is
(i) even (ii) a multiple of 3 (iii) even or a multiple of 3.

6. A card is drawn at random from a pack of 52.
What is the probability that the card is
(i) a club (ii) a king (iii) a club or a king
(iv) a red card (v) a queen (vi) a red card or a queen?

7. A pair of dice are thrown. What is the probability of getting
(i) a total of 12
(ii) the same number on both dice
(iii) a total of 12 or the same number on both dice?

8. Martin picks a card at random from this set.



Martin says

> The probability of picking a yellow card is $\frac{2}{5}$.
> The probability of picking a 3 is $\frac{2}{5}$.
> So the probability of picking a yellow card or a 3 is $\frac{2}{5} + \frac{2}{5} = \frac{4}{5}$.

(i) Explain why Martin is wrong.
(ii) What is the correct probability of picking a yellow card or a 3?

9. An ordinary dice is rolled.
Explain whether or not these pairs of outcomes are mutually exclusive.
The first one is done for you.

	First outcome	Second outcome
(i)	The score is 5.	The score is 3.
	These outcomes are mutually exclusive. A dice cannot show a score of 3 and a score of 5 at the same time.	
(ii)	The score is 3.	The score is an even number.
(iii)	The score is an even number.	The score is greater than 4.
(iv)	The score is a prime number.	The score is an even number.
(v)	The score is a multiple of 5.	The score is a multiple of 3.