Revision Exercise (Extended-Response Questions)

1. A person who contracts a particular disease requires treatment with a certain drug. The concentration C of that drug in the bloodstream, t hours after taking a dose of the drug, is given by the equation $C(t) = 0.02t - at^3$. The concentration C is measured as 0.075, five hours after taking the first dose.

(i) Find the value of the constant a.
(ii) For how many hours is some of the drug still in the bloodstream?
(iii) Explain why the graph of $C(t)$ is approximately linear up to $t = 10$ hours.

(i) $C(5) = 0.02(5) - a(5)^3 = 0.075$

$\Rightarrow 0.1 - 125a = 0.075$

$a = \dfrac{-0.1 + 0.075}{-125} = 0.0002$

$\Rightarrow C(t) = 0.02t - 0.0002t^3$

(iii)
If a cubic graph of shape $y = bx - ax^3$ appears linear it is because its behaving like the ax^3 part is small in value.

(ii) $0.2t - 0.0002t^3 = 0$

It's neater if you multiply both sides by 5000

$100t - 1t^3 = 0$

$t(100 - t^2) = 0$

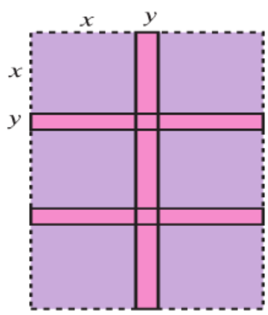
$t = 0$ | $100 - t^2 = 0$, $t^2 = 100$, $t = \pm 10$

we have 3 possible answers

but only $t = 10$ hours makes sense!

2. A large rectangular poster is subdivided into 6 purple squares of side x m, with dividing strips y m wide as shown.

(i) Find the area of the full poster in terms of x and y.
(ii) If the area of the dividing strips can be written in the form $kxy + 2y^2$, find k.
(iii) If the total area of the purple is 1.5 m^2, and the area of the dividing strips is 1 m^2, find x and hence find an equation for y and solve it.



(i)
Area = length × width

$A = (3x + 2y)(2x + y)$

$= 6x^2 + 3xy + 4xy + 2y^2$

$= 6x^2 + 7xy + 2y^2$

(ii)
$kxy + 2y^2 = $ Area Strips

$= 4xy + y(3x + 2y)$

$= 4xy + 3xy + 2y^2$

$= 7xy + 2y^2$

$\Rightarrow k = 7$

(iii) Area Strips $= 7xy + 2y^2 = 1$

Area purple $= 1.5$

$6x^2 = 1.5$

$x^2 = 1.5/6 = \frac{1}{4}$

$x = \frac{1}{2}$

$\Rightarrow 7(\tfrac{1}{2})y + 2y^2 - 1 = 0$

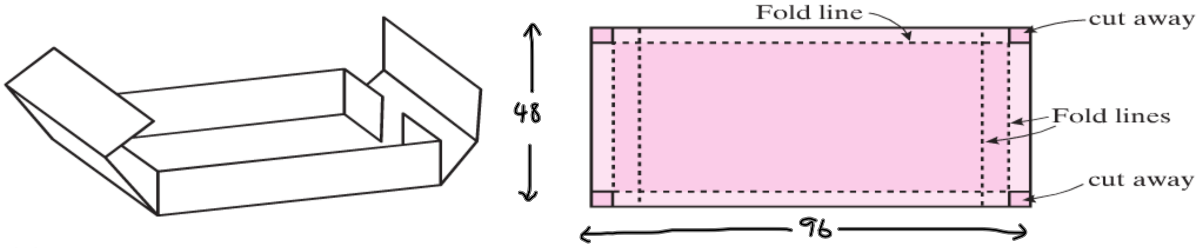
(x2) $4y^2 + 7y - 2 = 0$

$(4y - 1)(y + 2) = 0$

$y = \frac{1}{4}$ | $y = -2$ [doesn't make sense]

3. A TY project consists of making a reinforced box, as shown in diagram. The plan for the box is as follows:

- Squares of side x cm are cut from the four corners of a rectangular piece of cardboard that measures 48 cm by 96 cm.
- The fold lines are indicated by dotted lines.
- Two flaps are then folded with a double thickness of card at each end.



(a) Find an expression for the volume V of the open box.

$V = L \times B \times H$

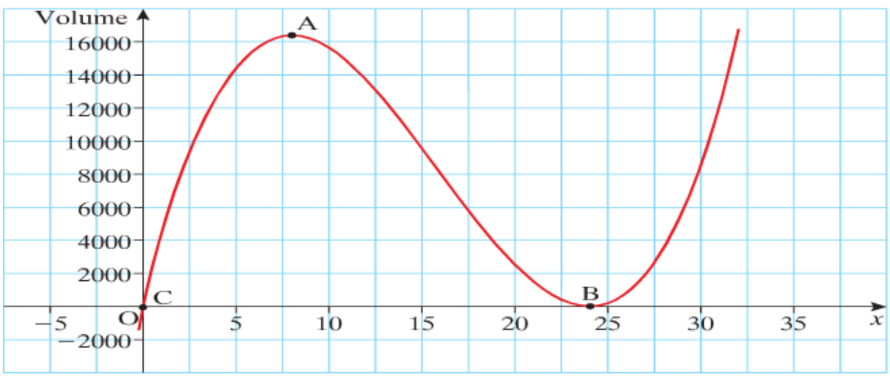
$L = 96 - 4x$

$B = 48 - 2x$

$H = x$

$V = (96-4x)(48-2x)(x)$

(b) A section of the graph of this expression is given in the diagram shown.



(i) What domain set of values of x are valid for making this box?
(ii) Explain the significance of the points A, B and C.
(iii) Estimate from the graph the maximum volume of the box and the value of x at which this occurs.
(iv) Find the volume of the box when $x = 10$ cm.
(v) It is decided that $0 < x < 5$ cm. Find the maximum volume possible.
(vi) If $5 \leqslant x \leqslant 15$ cm, what is the minimum volume of the box?

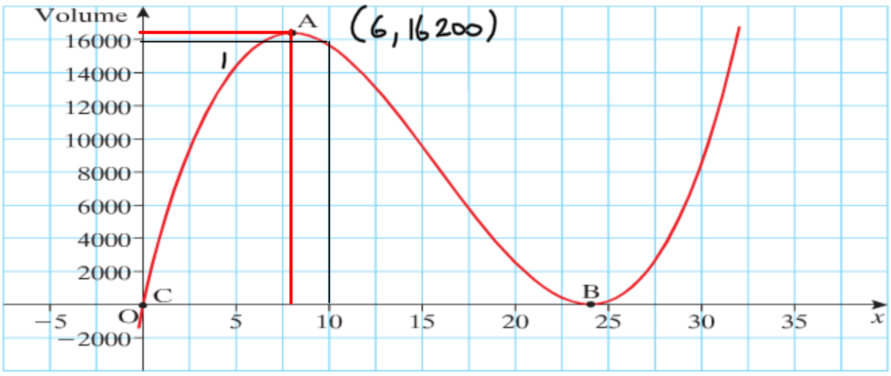
(i) Domain = ? For height to be greater than $0 \Rightarrow x > 0$

For width to be greater than $0 \Rightarrow 48 - 2x > 0 \Rightarrow x < 24$

Domain : $0 < x < 24$

(ii) B and C are at the limits of possible x values, the box would have no volume if $x = 0$ or 24. A gives the max volume of the box.

(b) A section of the graph of this expression is given in the diagram shown.



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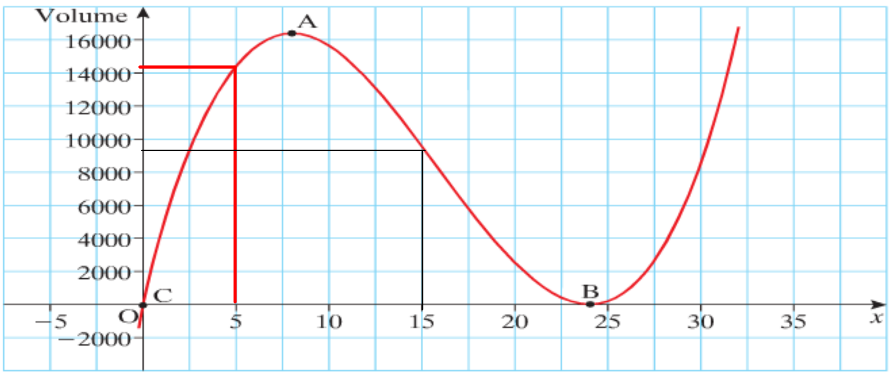
(iii) estimate: max volume is 16200 cm^3 when $x = 6$ cm

(iv) $x=10$, $V = (96-4x)(48-2x)(x)$
$= (96-4(10))(48-2(10))(10) = (56)(28)(10) = 15{,}680\ cm^3$

(b) A section of the graph of this expression is given in the diagram shown.



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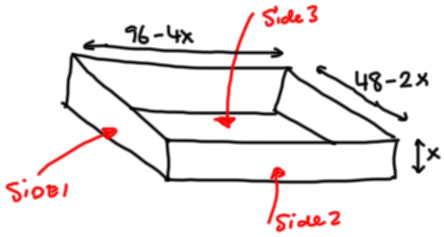
(v) max when $0 < x < 5$ occurs when $x=5$

$\Rightarrow V = (96-4(5))(48-2(5))(5) = 76(38)(5) = 14{,}440\ cm^3$

(vi) min when $5 < x < 15$ occurs when $x=15$

$\Rightarrow V = (96-4(15))(48-2(15))(15) = 36(18)(15) = 9{,}720\ cm^3$

(c) The external surface area of the box can be given by the formula $A = a(b - x)(c + x)$; find the values of a, b and c.



external Surface

$= (\text{Side } 1) \times 2$

$+ (\text{Side } 2) \times 2$

$+ (\text{Side } 3) \times 1$

$$A = (2(x)(48-2x)) + (2(x)(96-4x) + (48-2x)(96-4x)$$

$$= 2x(48-2x+96-4x) + 4608 - 192x - 192x + 8x^2$$

$$= 2x(144-6x) + 4608 - 384x + 8x^2$$

$$= 288x - 12x^2 + 4608 - 384x + 8x^2 = -4x^2 - 96x + 4608$$

$$= -4[x^2 + 24x - 1152] = -4(x+48)(x-24)$$

4. Riding stables need temporary additional paddock space for an upcoming horse show. There is sufficient funding to rent 120 m of temporary chain-link fencing. The plan is to form two paddocks with a shared fence running down the middle.

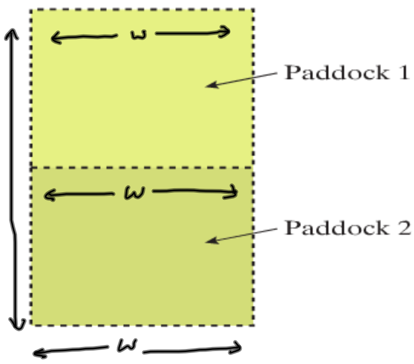
(i) Show that the area of the paddocks can be represented by the quadratic equation $A = -\frac{3}{2}w^2 + 60w$, where A stands for the area and w for the width of the paddock.

(ii) Find the roots of this equation and hence draw a rough sketch of the curve.

(iii) By completing the square of the equation for the area A, find the maximum area of the paddock, and

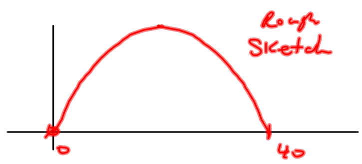
(iv) the value of w at which this maximum area occurs.

(v) Hence find the dimensions of each of the paddocks.



(i) $\text{Area} = L \times W$ $L = \frac{120 - 3w}{2} = 60 - \frac{3}{2}w$

$\Rightarrow A = (60 - \frac{3}{2}w)w = \boxed{60w - \frac{3}{2}w^2}$



(ii) $-\frac{3}{2}w^2 + 60w = 0 \Rightarrow w(-\frac{3}{2}w + 60) = 0 \Rightarrow w = 0$ OR $w = \frac{-60}{(-3/2)} = 40\text{m}$

(iii) $A = -\frac{3}{2}w^2 + 60w = -\frac{3}{2}\left[w^2 - 40w\right] =$

	w	-20
w	w^2	$-20w$
-20	$-20w$	400

$= -\frac{3}{2}\left[w^2 - 40w + 400 - 400\right]$

$= -\frac{3}{2}\left[(w-20)^2 - 400\right]$

$= -\frac{3}{2}(w-20)^2 + 600$

max area $= 600\ m^2$

(iv) width at max area $= 20$ m

(v) If $w = 20$m $\quad L = 60 - \frac{3}{2}(20) = 60 - 30 = 30$ m

So each paddock is: $20\text{m} \times 15\text{m}$

5. A golf ball is hit from the top of a 2 m-high tee. If the height of the ball, h, above ground level is given by the equation $h = 2 + 4t - t^2$, where t is the time measured in seconds,

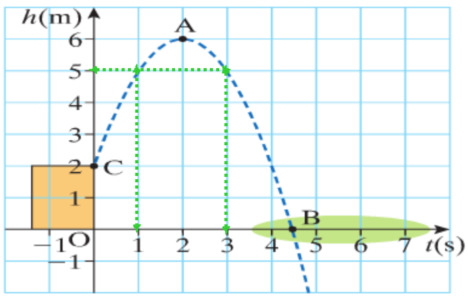
(a) estimate from the graph:

(i) the times, t, at which the ball is 5 m above ground level,

(ii) the time the ball takes to land on the ground.

(b) Find, correct to two places of decimals, the time taken by the ball to reach the ground.

(c) The equation $h = 2 + 4t - t^2$ can be written in the form $q - (t - p)^2$ for all values of t, where q is the highest point of the ball above ground, at time p. Find (p, q).



(a) (i) at 1s and 3s $\quad$ (ii) 4.5 s

(b) when ball hits ground $h=0$ $\Rightarrow$ $2 + 4t - t^2 = 0$ $\Rightarrow$ $t^2 - 4t - 2 = 0$

$a=1$
$b=-4$
$c=-2$

$$t = \frac{+4 \pm \sqrt{(-4)^2 - 4(1)(-2)}}{2(1)} = \frac{4 \pm \sqrt{24}}{2} = 2 \pm \sqrt{6} \simeq 4.45\ s$$

(only positive time makes sense here)

$h = 2 + 4t - t^2$ Write in form $q - (t - p)^2$

$$h = -t^2 + 4t + 2 = -1\left[t^2 - 4t - 2\right]$$
$$= -1\left[t^2 - 4t + 4 - 2 - 4\right]$$
$$= -1\left[(t-2)^2 - 6\right]$$
$$= -(t-2)^2 + 6$$
$$h = 6 - (t-2)^2$$

	t	-2
t	t^2	$-2t$
-2	$-2t$	$+4$

$\Rightarrow$ $q = 6$ and $p = 2$

6. You have managed a bike-rental scheme in a seaside holiday resort for the summer. You found that if you charged €12.00 per bike per day, then on average you did 36 rentals per day.
For every 50 cent increase in the rental price, the average number of rentals decreased by 2 rentals per day.
Complete the following table.

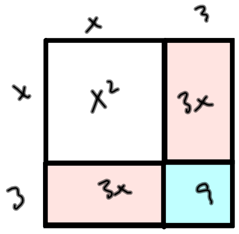
No. of price hikes	Price per rental	Number of rentals	Total income (I)
	€12	36	€ 432
1 price hike	€12·50	34	€ 425
2 price hikes	€ 13	32	€ 416
3 price hikes	€ 13·50	30	€ 405
x price hikes	€ $12 + 0{\cdot}5x$	$36 - 2x$	€ $432 - 6x - x^2$

$$(12 + 0{\cdot}5x)(36 - 2x) = 432 - 24x + 18x - x^2$$
$$= 432 - 6x - x^2$$

(i) Write an equation in terms of x for the income I.
(ii) Write this equation in the form $q-(x-p)^2$, where (p, q) is the maximum point of the curve.
(iii) Use this information to find the maximum income.
(iv) What should you change to increase income?

(i) $I = 432 - 6x - x^2$

(ii) $I = -1[x^2 + 6x - 432] = -1[x^2 + 6x + 9 - 9 - 432]$



$= -1[(x+3)^2 - 441]$

$= -(x+3)^2 + 441$

$I = 441 - (x+3)^2$

(iii) max I = €441

(iv) $x = -3$ means price should be reduced by €1·50 to €10·50