Sequences – Series – Patterns

Chapter 4

Key words

number sequence arithmetic sequence series sigma (Σ)
geometric sequence exponential sequence geometric series recurring decimal
finite difference composite function quadratic function

Example 1

Write down the first four terms of each of the following sequences:

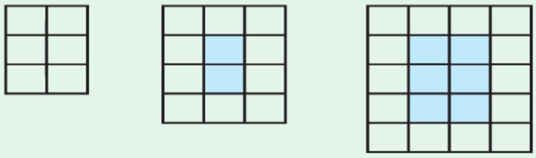
(i) $T_n = n^2 + n$ (ii) $T_n = 2^n - 3n$

(i)
$T_n = n^2 + n$
$T_1 = (1)^2 + 1 = 2$
$T_2 = (2)^2 + 2 = 6$
$T_3 = (3)^2 + 3 = 12$
$T_4 = (4)^2 + 4 = 20$

(ii)
$T_n = 2^n - 3n$
$T_1 = 2^1 - 3(1) = 2 - 3 = -$
$T_2 = 2^2 - 3(2) = 4 - 6 = -2$
$T_3 = 2^3 - 3(3) = 8 - 9 = -$
$T_4 = 2^4 - 3(4) = 16 - 12 = -$

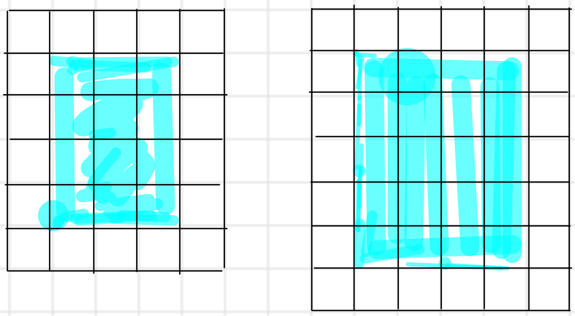
Example 2

The following rectangular patterns are made from two sets of coloured tiles.



(i) Draw the next two patterns of tiles.
(ii) Write a number sequence for the blue tiles used in each of these patterns.
(iii) Write a number sequence for the total number of tiles used in each of these patterns.
(iv) Write a number sequence for the white tiles used in each of these patterns.
(v) Write out the next 3 terms in each sequence found in (ii), (iii), (iv).

(i)



(ii) Blue: 0, 2, 6, 12, 20 . . . 30, 42, 56
(v) (iii) All tiles: 6, 12, 20, 30, 42 . . . 56, 72, 90
(iv) White tiles: 6, 10, 14, 18, 22 . . . 26, 30, 34

Exercise 4.1

1. Write down the next three terms of each of the following sequences:

quadratic +4, +6, +8... (i) 6, 12, 18, 24, . . . 34, 46, 60
arithmetic +5 (ii) 7, 12, 17, 22, . . . 27, 32, 37
+1·2 (iii) 4.7, 5.9, 7.1, 8.3, . . . 9.5, 10.7, 11.9
−3 (iv) 2, −1, −4, −7, . . . −10, −13, −16
quadratic +1, +3, +5... (v) 2, 3, 6, 11, 18, 27, . . . 38, 50, 73

arithmetic −8 (vi) 78, 70, 62, 54, . . . 46, 38, 30
arithmetic −5 (vii) 10, 5, 0, −5, −10, . . . −15, −20, −25
arithmetic −9 (viii) −64, −55, −46, −37, . . . −28, −19, −10
geometric ×3 (ix) 2, 6, 18, . . . 54, 162, 486
quadratic +4, +6, +8... (x) 2, 6, 12, 20, . . . 30, 42, 56

arithmetic $-\frac{1}{2}$ (xi) $\frac{3}{4}$, $\frac{1}{4}$, $-\frac{1}{4}$, $-\frac{3}{4}$, $-\frac{5}{4}$, $-\frac{7}{4}$
quadratic +1, +2, +3... (xii) 1, 2, 4, 7, 11, . . . 16, 22, 29
quadratic +3 +5, +7.. (xiii) 0, 3, 8, 15, 24, . . . 35, 48, 63
geometric x−2 (xiv) 3, −6, 12, −24, . . . 48, −96, 192
quadratic (inverse) (xv) $\frac{1}{2}$, $\frac{1}{6}$, $\frac{1}{12}$, $\frac{1}{20}$, . . . $\frac{1}{30}$, $\frac{1}{42}$, $\frac{1}{56}$

2. Find the first four terms of the following sequences, given the nth term (T_n) in each case.

(i) $T_n = 4n - 2$ (iv) $T_n = (n+3)(n+1)$ (vii) $T_n = 2^n$

(ii) $T_n = (n+1)^2$ (v) $T_n = n^3 - 1$ (viii) $T_n = (-3)^n$

(iii) $T_n = n^2 - 2n$ (vi) $T_n = \frac{n}{n+2}$ (ix) $T_n = n.2^n$

(i) $T_n = 4n-2$
$T_1 = 4(1)-2 = 2$
$T_2 = 4(2)-2 = 6$
$T_3 = 4(3)-2 = 10$
$T_4 = 4(4)-2 = 14$

(ii) $T_n = (n+1)^2$
$T_1 = (1+1)^2 = 4$
$T_2 = (2+1)^2 = 9$
$T_3 = (3+1)^2 = 16$
$T_4 = (4+1)^2 = 25$

(iii) $T_n = n^2 - 2n$
$T_1 = (1)^2 - 2(1) = -1$
$T_2 = (2)^2 - 2(2) = 0$
$T_3 = (3)^2 - 2(3) = -3$
$T_4 = (4)^2 - 2(4) = 8$

(iv) $T_n = (n+3)(n+1)$
$T_1 = (1+3)(1+1) = 8$
$T_2 = (2+3)(2+1) = 15$
$T_3 = (3+3)(3+1) = 36$
$T_4 = (4+3)(4+1) = 60$

(v) $T_n = n^3 - 1$
$T_1 = (1)^3 - 1$
$T_2 = (2)^3 - 1$
$T_3 = (3)^3 - 1$
$T_4 = (4)^3 - 1$

(vi) $T_n = n/(n+2)$
$T_1 = 1/(1+2) = 1/3$
$T_2 = 2/(2+2) = 1/2$
$T_3 = 3/(3+2) = 3/5$
$T_4 = 4/(4+2) = 2/3$

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(iii) $T_n = n^2 - 2n$ (vi) $T_n = \frac{n}{n+2}$ (ix) $T_n = n.2^n$

(vii) $T_n = 2^n$
$T_1 = 2^1 = 2$
$T_2 = 2^2 = 4$
$T_3 = 2^3 = 8$
$T_4 = 2^4 = 16$

(viii) $T_n = (-3)^n$
$T_1 = (-3)^1 = -3$
$T_2 = (-3)^2 = 9$
$T_3 = (-3)^3 = -27$
$T_4 = (-3)^4 = 81$

(ix) $T_n = n.2^n$
$T_1 = (1)2^1 = 2$
$T_2 = (2)2^2 = 8$
$T_3 = (3)2^3 = 24$
$T_4 = (4)2^4 = 64$