**12.** Show that the sum of the natural numbers from 1 to $n$ is $\frac{n}{2}(n+1)$ and use the formula to find the sum of $1+2+3+4+\ldots..99$.

$1+2+3+\ldots.+n$

$a = 1, \quad d = 1, \quad n = n$

$S_n = \frac{n}{2}[2a+(n-1)d]$

$S_n = \frac{n}{2}[2(1)+(n-1)1]$

$S_n = \frac{n}{2}[2+n-1]$

$S_n = \frac{n}{2}(n+1)$

$S_{99} = \frac{99}{2}(99+1) = 4950$ ✓

**14.** In an arithmetic sequence, $T_{21} = 37$ and $S_{20} = 320$. Find the sum of the first ten terms.

$T_n = a+(n-1)d$

$T_{21} = 37$

$\Rightarrow 37 = a+(21-1)d$

$a + 20d = 37$ ①

$S_n = \frac{n}{2}[2a+(n-1)d]$

$S_{20} = 320$

$\Rightarrow 320 = \frac{20}{2}[2a+(20-1)d]$

$32 = 2a+19d$ ②

2①−② $\Rightarrow$ $\cancel{2a} + 40d = 74$

$-\cancel{2a} - 19d = 32$

$21d = 42 \Rightarrow d = 2$ ③

③ → ① $\Rightarrow a + 20(2) = 37 \Rightarrow a = -3$

$S_n = \frac{n}{2}[2a+(n-1)d]$

$S_{10} = \frac{10}{2}[2(-3)+(10-1)2]$

$= 5[-6+18] = 5[12] = 60$ ✓

**15.** Show that $S_n = \frac{n(a+l)}{2}$ is the sum to $n$ terms of an arithmetic sequence where $l$ is the last term.

$l = \quad T_n = a + (n-1)d$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$= \frac{n}{2}\left[a + (a + (n-1)d)\right]$$

$$= \frac{n}{2}\left[a + l\right] \quad ✓$$

**16.** Explain why $S_\infty$ (the sum to infinity) for an arithmetic sequence cannot be found.

$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$\Rightarrow$ "$S_\infty = \frac{\infty}{2}\left[2a + (\infty - 1)d\right]$"

this is problematic because
$\infty/2$ makes no sense
as does $(\infty - 1)$
as does $(\infty - 1)d$
etc....

$\Rightarrow$ to sum you need an end term and this is not the case in $S_\infty$