Note:

- Given three consecutive terms of a geometric sequence, T_1, T_2, T_3, we note that $\frac{T_2}{T_1} = \frac{T_3}{T_2} = $ (common ratio, r).
- We also note that $\frac{a}{r}, a, ar$ are three consecutive terms of a geometric sequence, with first term $\frac{a}{r}$ and common ratio r.

  Multiplying these terms gives $\frac{a}{\cancel{r}} \times a \times a\cancel{r} = a^3$, i.e. the cube of the middle term.

  For example: $2, 6, 18$ are in geometric sequence,

  $\Rightarrow \quad 2 \times 6 \times 18 = 216 = 6^3$

Example 3

$3, x, x + 6, \ldots$ are the first three terms of a geometric sequence of positive terms.

Find

(i) the value of x (ii) the tenth term of the sequence.

$\frac{T_2}{T_1} = \frac{T_3}{T_2}$

$\frac{x}{3} = \frac{x+6}{x} \Rightarrow x^2 = 3x + 18$

$x^2 - 3x - 18 = 0$

$(x-6)(x+3) = 0$

Positive terms

$x = 6$ ✓, $x = -3$ ✗

Sequence: $3, 6, 12 \ldots$ $a = 3$, $r = 2$

$T_n = ar^{n-1}$

$T_{10} = 3(2)^9 = 1536$

$$\frac{T_3}{T_2} = r$$

$$\frac{T_4}{T_2} = r^2$$

etc..

$T_2 . T_3 , T_4 . T_5$

$T_2 = 4$

$T_5 = -\frac{1}{16}$

$4r^3 = -\frac{1}{16}$

$$\Rightarrow \frac{\left(-\frac{1}{16}\right)}{4} = r^3$$

$$r = \sqrt[3]{\left(\frac{\left(-\frac{1}{16}\right)}{4}\right)} = -\frac{1}{4}$$

9. The first term of a geometric sequence is 16 and the fifth term is 9. What is the value of the seventh term?

$\sqrt{r^4} = \sqrt{\frac{9}{16}}$

$r^2 = \frac{3}{4}$

$r = \sqrt{3/4}$

$T_n = ar^{n-1}$

$T_1 = a = 16$

$T_5 = 9$

$\Rightarrow 16r^4 = 9$

$r^4 = \frac{9}{16}$; $r = \sqrt[4]{\frac{9}{16}} = \sqrt{\sqrt{\frac{9}{16}}} = \sqrt{\frac{3}{4}}$

$T_7 = 16\left(\sqrt{\frac{3}{4}}\right)^6 = 6.75$ ✓

$\underset{\times r}{\frown}\ \underset{\times r}{\frown}\ \underset{\times r}{\frown}\ \underset{\times r}{\frown}$

T_1, T_2, T_3, T_4, T_5

alternative method

9. The first term of a geometric sequence is 16 and the fifth term is 9. What is the value of the seventh term?

$T_n = ar^{n-1}$

$T_1 = a = 16$

$T_5 = 9$

$9 = 16(r)^4$

$r^4 = \frac{9}{16} \Rightarrow r = \sqrt{\frac{3}{4}}$

$T_n = ar^{n-1}$

$T_7 = 16\left(\sqrt{\frac{3}{4}}\right)^6 = 6.75$

Example 4

The product of the first three terms of a geometric sequence is 216 and their sum is 21. Given that the common ratio r is less than 1, find the first three terms of the sequence.

let terms = $a,\ ar,\ ar^2$

$a \times ar \times ar^2 = a^3r^3 = 216$

$a^3r^3 = 216 \Rightarrow a = \sqrt[3]{\frac{216}{r^3}} = \frac{6}{r}$

$a + ar + ar^2 = 21$

$\Rightarrow \frac{6}{r} + \frac{6}{r}r + \frac{6}{r}r^2 = 21$

$\times r$

$6 + 6r + 6r^2 = 21r$

Solve

$6r^2 - 15r + 6 = 0$

$2r^2 - 5r + 2 = 0$

$(2r-1)(r-2)$

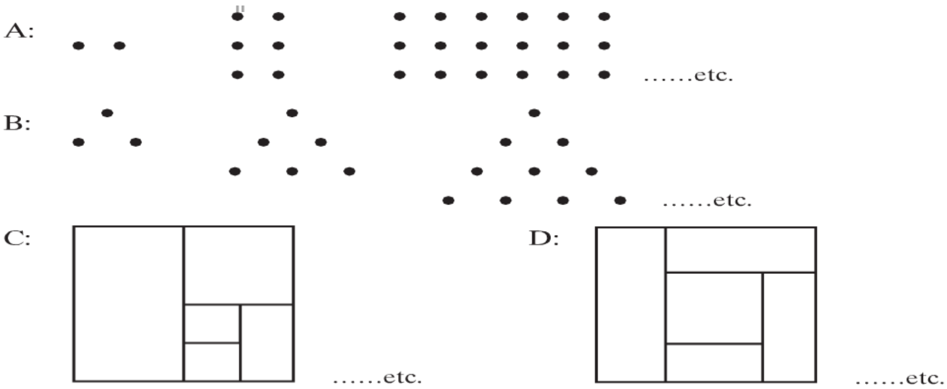
$r<1$

$r = \frac{1}{2}$ ✓ , $r = 2$ ✗

$a = \frac{6}{(\frac{1}{2})} \Rightarrow a = 12$

Sequence = $12,\ 6,\ 3 \ldots$

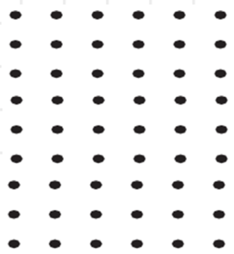
6.



By inspection, decide which of the above patterns generate a geometric sequence. Draw the next pattern of those that are geometric.

Only A is geometric $r = 3$

next term =



8. The third term of a geometric sequence is -63 and the fourth term is 189. Find
(i) the values of a and r
(ii) an expression for T_n.

(i)

$T_3 = -63$
$T_4 = 189$

$r = \frac{T_4}{T_3}$ $\qquad r = \frac{189}{-63} \Rightarrow r = -3$

$a = ?$ $\quad T_n = ar^{n-1}$ $\qquad T_3 = -63$

$\Rightarrow -63 = a(-3)^2$
$-63 = 9a$
$a = \frac{-63}{9} \Rightarrow a = -7$

(ii) $\quad T_n = (-7)(-3)^{n-1}$

10. The product of the first three terms of a geometric sequence is 27 and their sum is 13.
Find the first four terms of the sequence.

let 1st 3 terns be $\quad a, ar, ar^2$

Product $\quad a \times ar \times ar^3 = (ar)^3 = 27$

$\Rightarrow ar = \sqrt[3]{27} = 3 \Rightarrow a = \frac{3}{r}$ ①

Sum $\quad a + ar + ar^2 = 13$ ②

Sub in ① → ② $\quad \frac{3}{r} + \left(\frac{3}{r}\right)r + \left(\frac{3}{r}\right)r^2 = 13$

×r $\quad 3 + 3r + 3r^2 = 13r$

Solve $\quad 3r^2 - 10r + 3 = 0$
$(3r - 1)(r - 3) = 0$
$r = 1/3$ or $r = 3$

Sub into ① $\quad a = \frac{3}{(\frac{1}{3})} = 9$ or $a = \frac{3}{3} = 1$

First TERMS $\quad 9, 3, 1, \frac{1}{3}$ or $1, 3, 9, 27$